CSx25: Digital Signal Processing
NCS224: Signals and Systems

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Digital Signal Processing – What?

- Signal
  - A signal is formally defined as "a function of one or more variables that conveys information on the nature of a physical phenomenon."
  - A signal, as the term implies, is a set of information or data.
- Signal Processing deals with the representation, transformation, and manipulation of signals and the information they contain.
- System
  - A signal is *applied to* a system as *input*, and the system *responds* to the signal by producing another signal called the *output*.

Study: Neso Academy- Signals and Systems

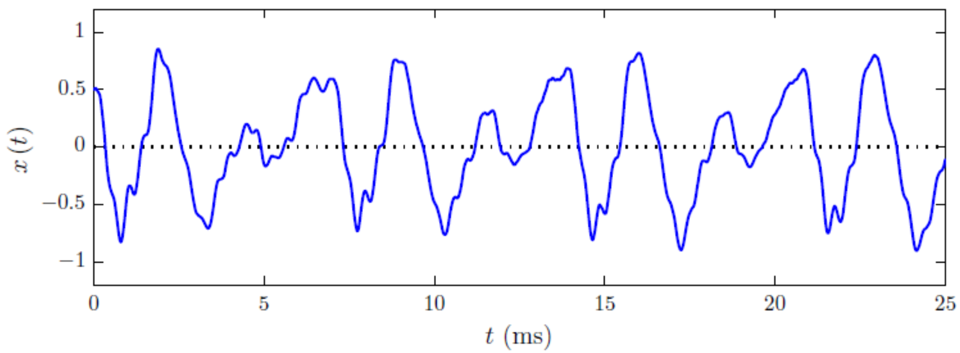
Outline

- Continuous Time Signals
- Discrete Time Signals

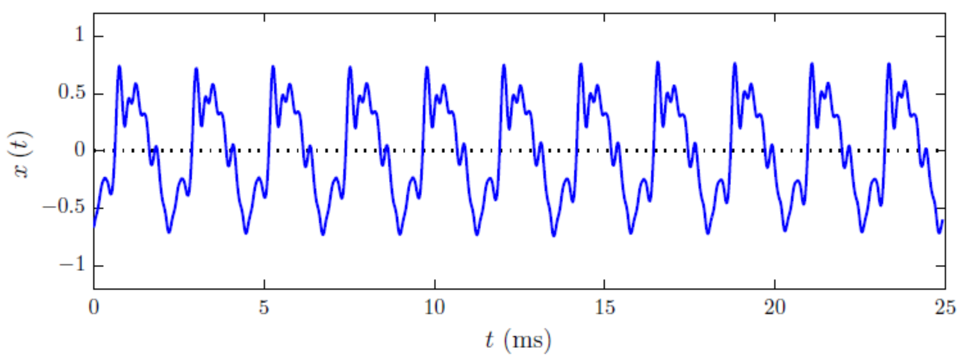
1.3 Continuous-Time Signals

Continuous-time signals

A segment from the vowel "o" of the word "hello"



A segment from the sound of a violin

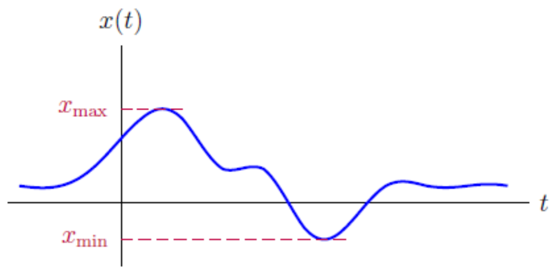


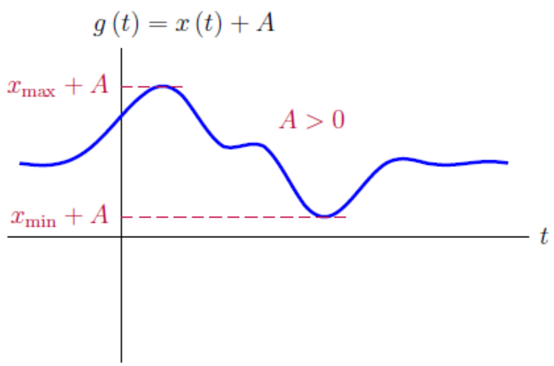
1.3 Continuous-Time Signals

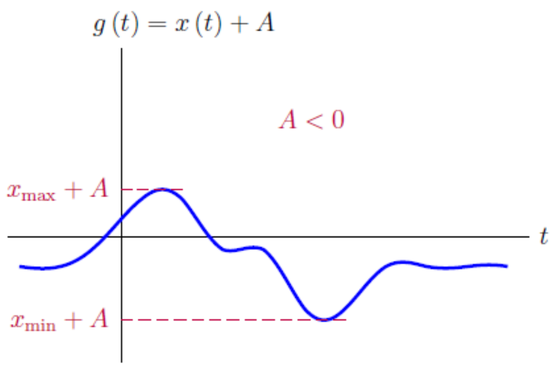
Signal operations

Addition of a constant offset

$$g(t) = x(t) + A$$





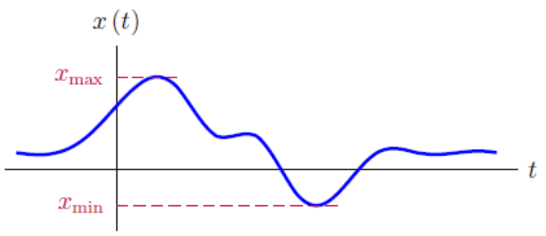


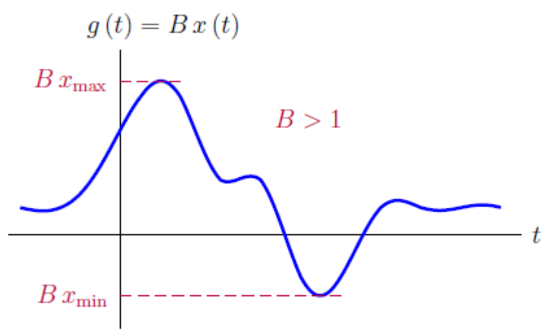
1.3 Continuous-Time Signals

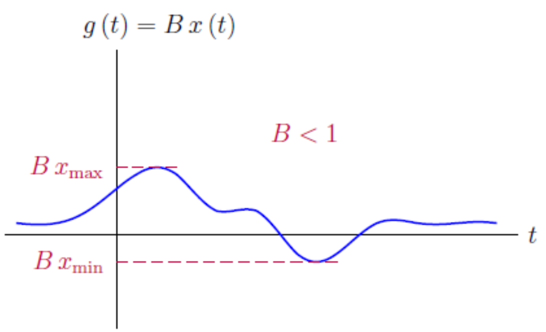
Signal operations (continued)

Multiplication by a constant gain factor

$$g(t) = B\,x(t)$$





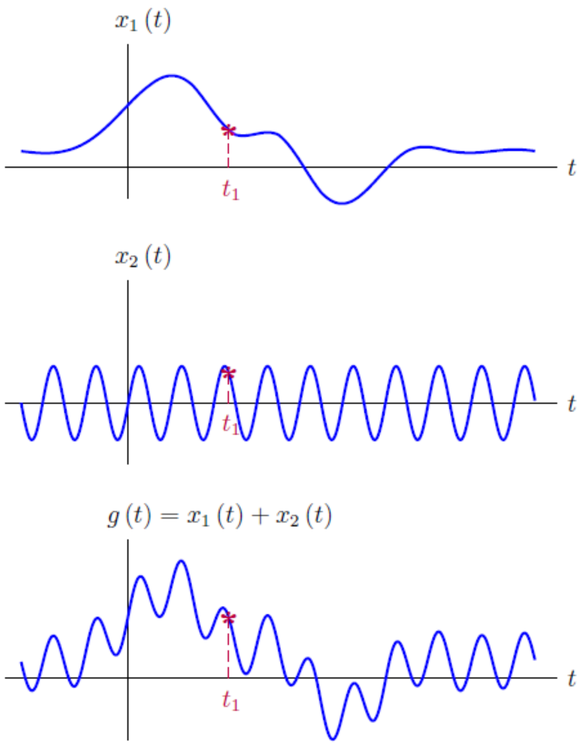


1.3 Continuous-Time Signals

Signal operations (continued)

Adding two signals

$$g(t) = x_1(t) + x_2(t)$$

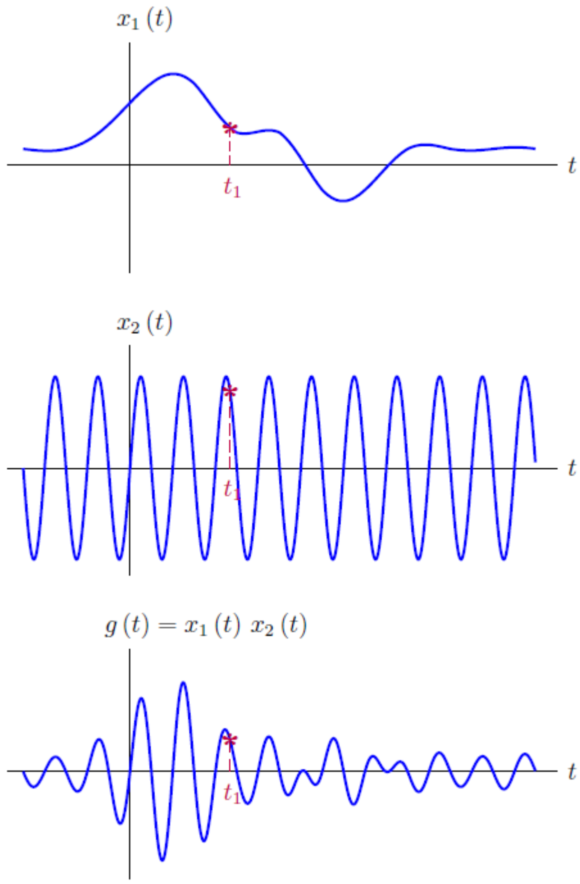


1.3 Continuous-Time Signals

Signal operations (continued)

Multiplying two signals

$$g(t) = x_1(t)\, x_2(t)$$



1.3 Continuous-Time Signals

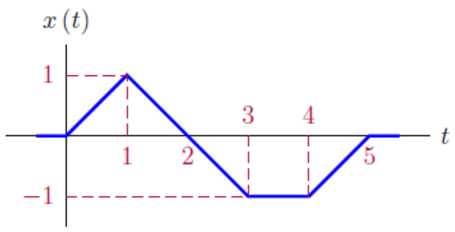
Example 1.1

Constant offset and gain

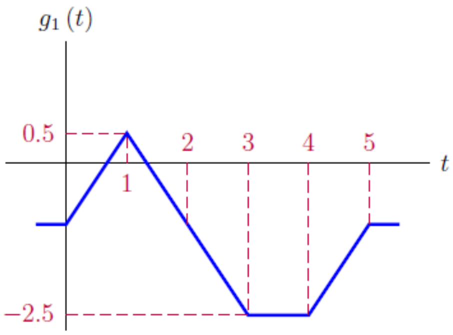
Consider the signal shown. Sketch the signals

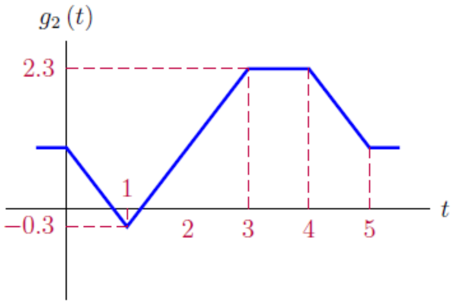
a. $g_1(t) = 1.5\,x(t) - 1$

b. $g_2(t) = -1.3\,x(t) + 1$



Solution:

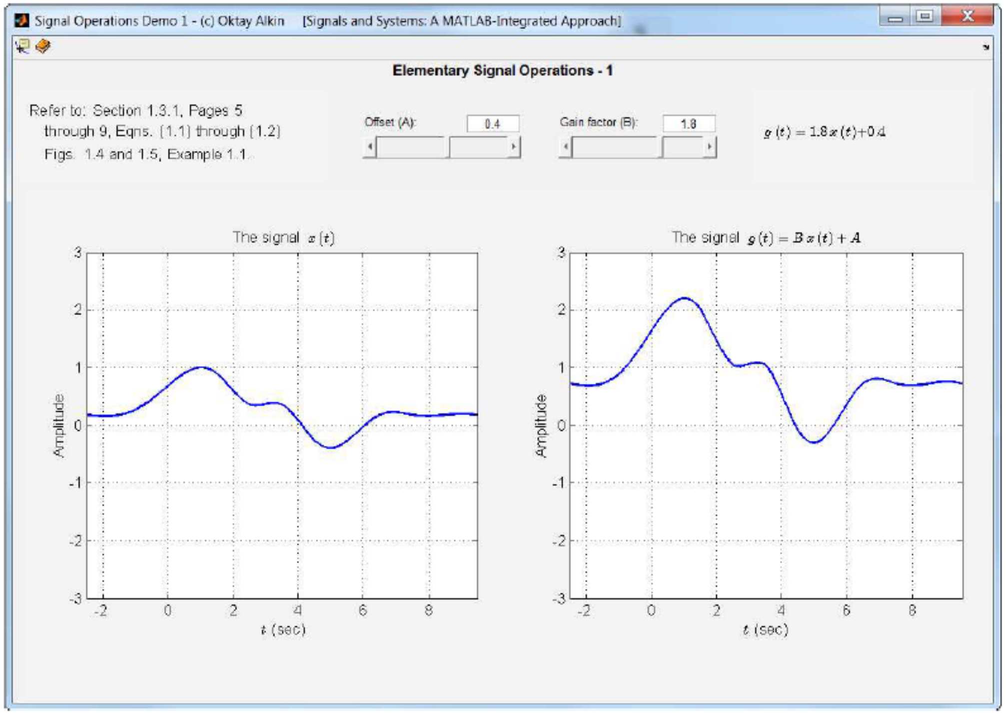




1.3 Continuous-Time Signals

Interactive demo: sop_demo1

Experiment by varying parameters A and B.



1.3 Continuous-Time Signals

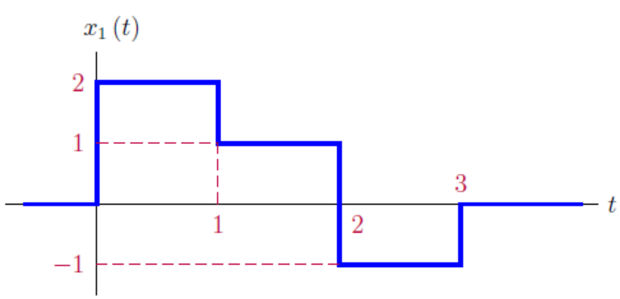
Example 1.2

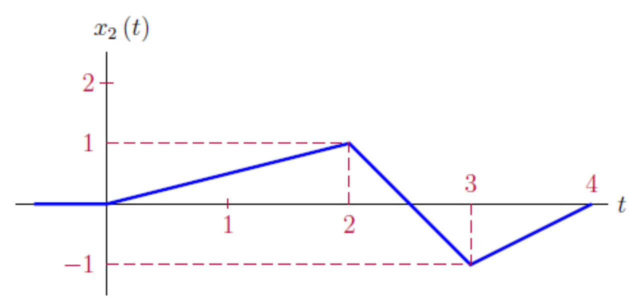
Arithmetic operations with continuous-time signals

Given the signals $x_1(t)$ and $x_2(t)$, sketch the signals

a. $g_1(t) = x_1(t) + x_2(t)$

b. $g_2(t) = x_1(t)\, x_2(t)$





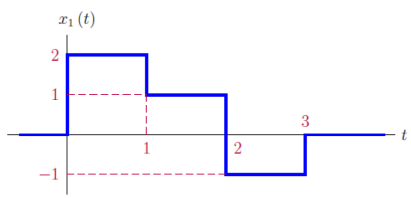
1.3 Continuous-Time Signals

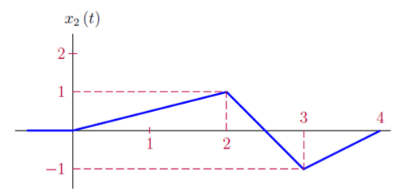
Example 1.2 (continued)

Solution - Part(a):

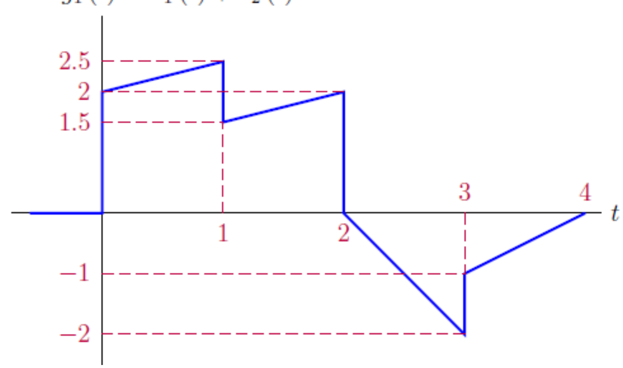
$$x_1(t) = \begin{cases} 2\,, & 0 < t < 1 \\ 1\,, & 1 < t < 2 \\ -1\,, & 2 < t < 3 \\ 0\,, & \text{otherwise} \end{cases}$$

$$x_2(t) = \begin{cases} \frac{1}{2}t\,, & 0 < t < 2 \\ -2t+5\,, & 2 < t < 3 \\ t-4\,, & 3 < t < 4 \\ 0\,, & \text{otherwise} \end{cases}$$





$g_1(t) = x_1(t) + x_2(t)$



$$g_1(t) = \begin{cases} \frac{1}{2}t+2\,, & 0 < t < 1 \\ \frac{1}{2}t+1\,, & 1 < t < 2 \\ -2t+4\,, & 2 < t < 3 \\ t-4\,, & 3 < t < 4 \\ 0\,, & \text{otherwise} \end{cases}$$

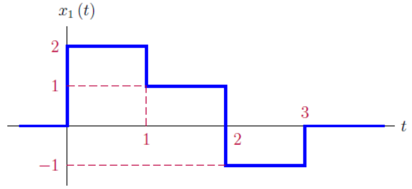
1.3 Continuous-Time Signals

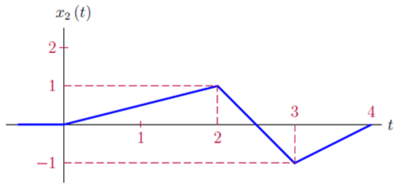
Example 1.2 (continued)

Solution - Part(b):

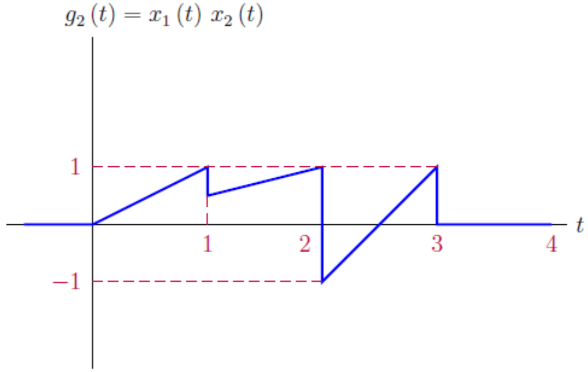
$$x_1(t) = \begin{cases} 2, & 0 < t < 1 \\ 1, & 1 < t < 2 \\ -1, & 2 < t < 3 \\ 0, & \text{otherwise} \end{cases}$$

$$x_2(t) = \begin{cases} \frac{1}{2}t, & 0 < t < 2 \\ -2t+5, & 2 < t < 3 \\ t-4, & 3 < t < 4 \\ 0, & \text{otherwise} \end{cases}$$





$$g_2(t) = \begin{cases} t, & 0 < t < 1 \\ \frac{1}{2}t, & 1 < t < 2 \\ 2t-5, & 2 < t < 3 \\ 0, & \text{otherwise} \end{cases}$$



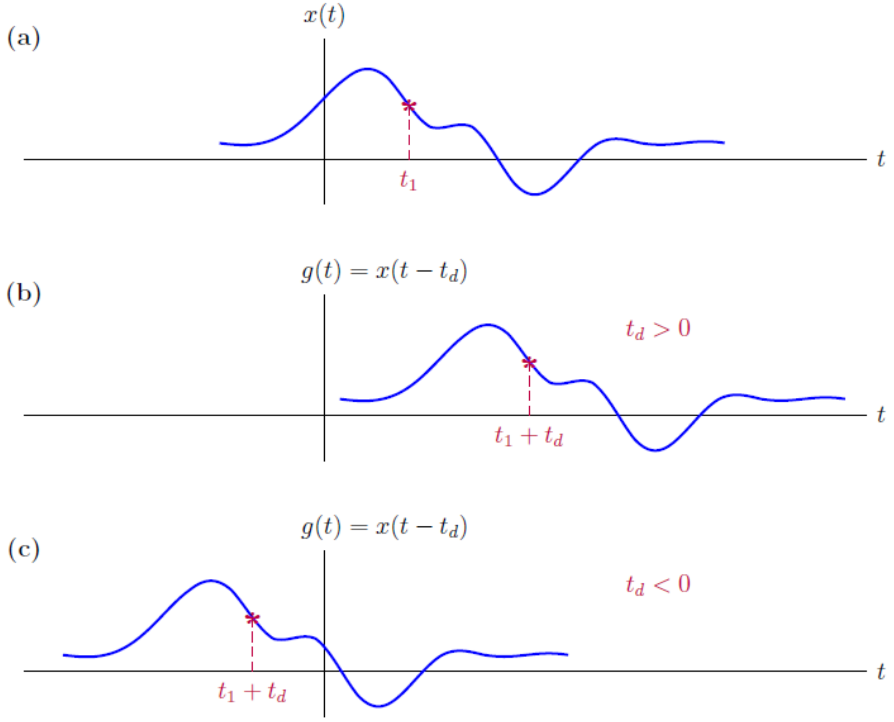
▸ MATLAB Exercise 1.2

1.3 Continuous-Time Signals

Signal operations (continued)

Time shifting

$$g(t) = x(t - t_d)$$

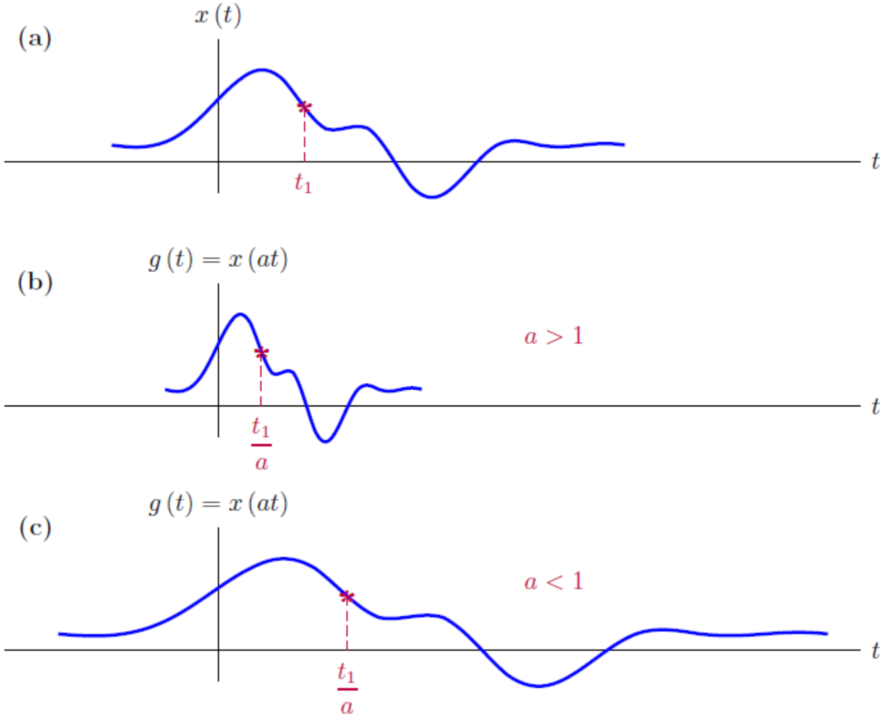


1.3 Continuous-Time Signals

Signal operations (continued)

Time scaling

$$g(t) = x(at)$$

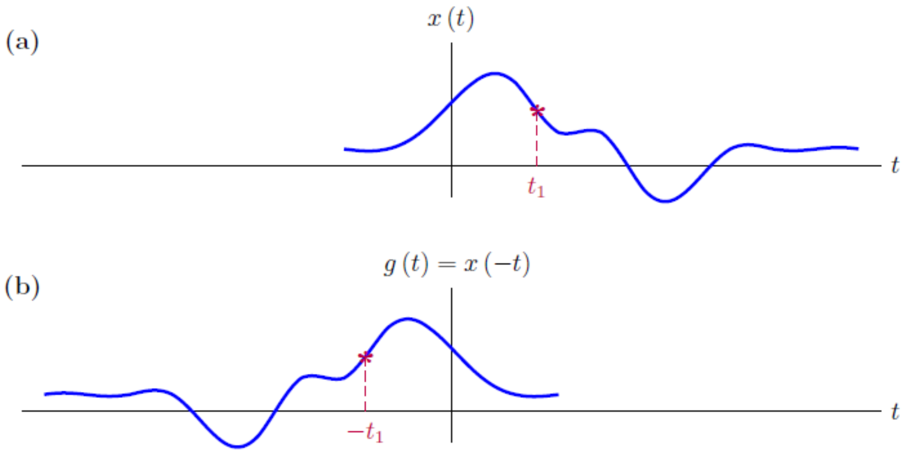


1.3 Continuous-Time Signals

Signal operations (continued)

Time reversal

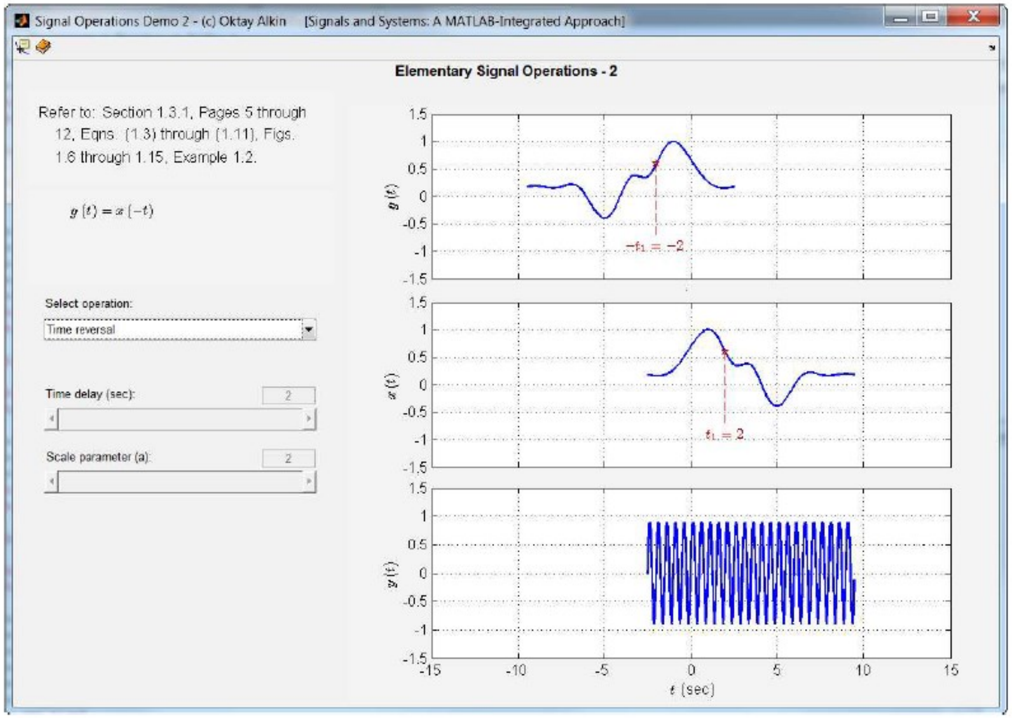
$$g(t) = x(-t)$$



1.3 Continuous-Time Signals

Interactive demo: sop_demo2

Experiment by varying applicable parameter values.



1.3 Continuous-Time Signals

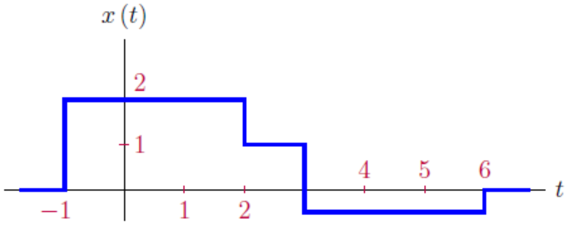
Example 1.3

Basic operations for continuous-time signals

Consider the signal $x(t)$ shown. Sketch the following signals:

a. $g(t) = x(2t - 5)$,

b. $h(t) = x(-4t + 2)$.

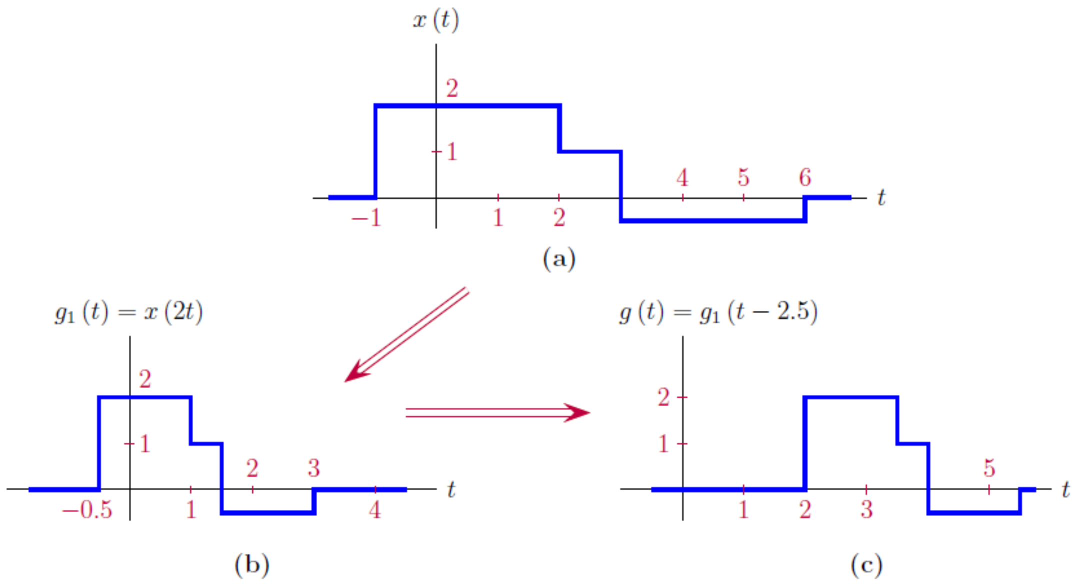


1.3 Continuous-Time Signals

Example 1.3 (continued)

Solution - Part(a):

$$g(t) = g_1(t - 2.5) = x(2[t - 2.5]) = x(2t - 5)$$



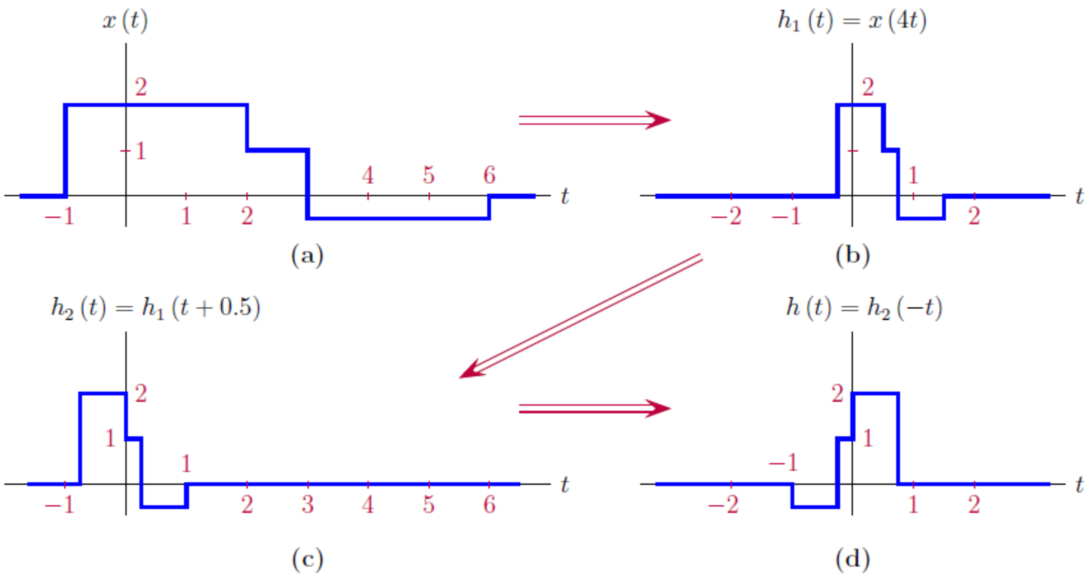
1.3 Continuous-Time Signals

Example 1.3 (continued)

Solution - Part(b):

$$h_2(t) = h_1(t+0.5) = x(4[t+0.5]) = x(4t+2)$$

$$h(t) = h_2(-t) = x(-4t+2)$$



MATLAB Exercise 1.3

1.3 Continuous-Time Signals

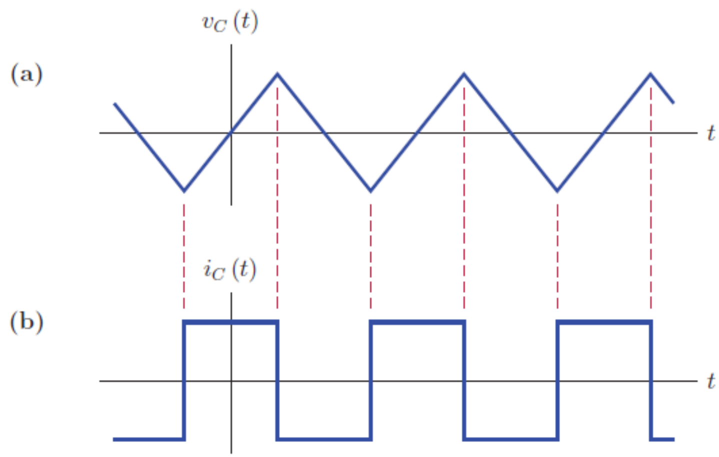
Integration and differentiation

Integration and differentiation operations are used extensively in the study of linear systems. Given a continuous-time signal $x(t)$, a new signal $g(t)$ may be defined as its time derivative in the form

$$g(t) = \frac{dx(t)}{dt} \tag{1.12}$$

A practical example of this is the relationship between the current $i_C(t)$ and the voltage $v_C(t)$ of an ideal capacitor with capacitance C as given by

$$i_C(t) = C\,\frac{dv_C(t)}{dt} \tag{1.13}$$



1.3 Continuous-Time Signals

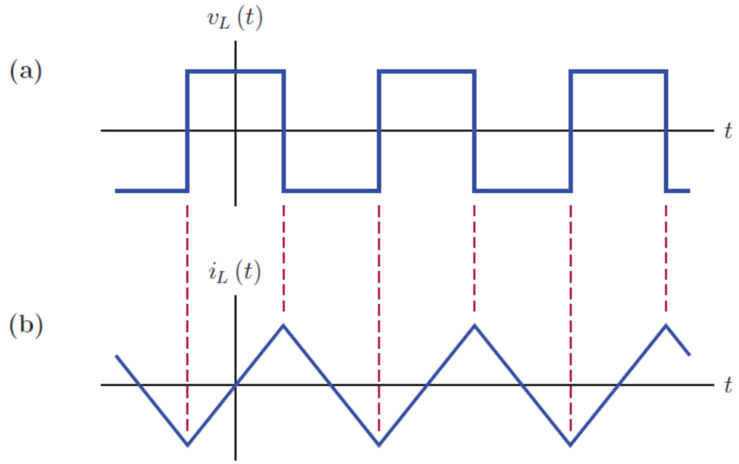
Integration and differentiation

Similarly, a signal can be defined as the integral of another signal in the form

$$g(t) = \int_{-\infty}^{t} x(\lambda)\, d\lambda \tag{1.14}$$

The relationship between the current $i_L(t)$ and the voltage $v_L(t)$ of an ideal inductor can serve as an example of this. Specifically we have

$$i_L(t) = \frac{1}{L} \int_{-\infty}^{t} v_L(\lambda)\, d\lambda \tag{1.15}$$



1.3 Continuous-Time Signals

Basic building blocks

- Unit-impulse function
- Unit-step function
- Unit-pulse function
- Unit-ramp function
- Unit-triangle function
- Sinusoidal signals

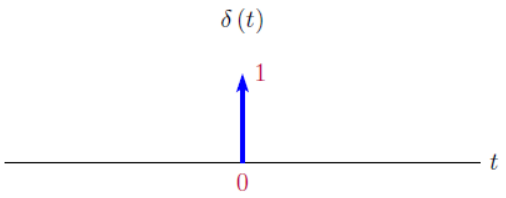
1.3 Continuous-Time Signals

Unit-impulse function

Mathematical definition

$$\delta(t) = \begin{cases} 0\,, & \text{if } t \neq 0 \\ \text{undefined}\,, & \text{if } t = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\; dt = 1$$



The value displayed next to the up arrow is not an amplitude value. Rather, it represents the area of the impulse function.

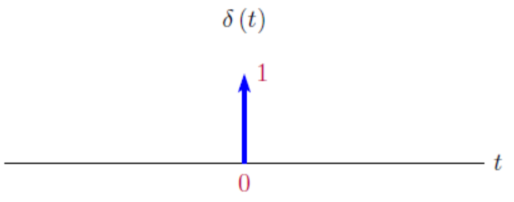
1.3 Continuous-Time Signals

Unit-impulse function

Mathematical definition

$$\delta(t) = \begin{cases} 0\ , & \text{if } t \neq 0 \\ \text{undefined}\ , & \text{if } t = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\ dt = 1$$

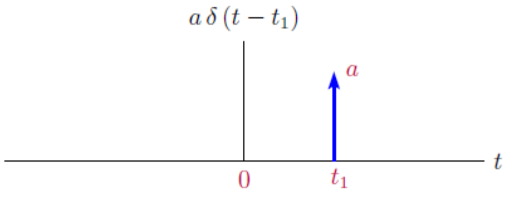


Scaling and time shifting:

$$a\,\delta(t - t_1) = \begin{cases} 0\ , & \text{if } t \neq t_1 \\ \text{undefined}\ , & \text{if } t = t_1 \end{cases}$$

and

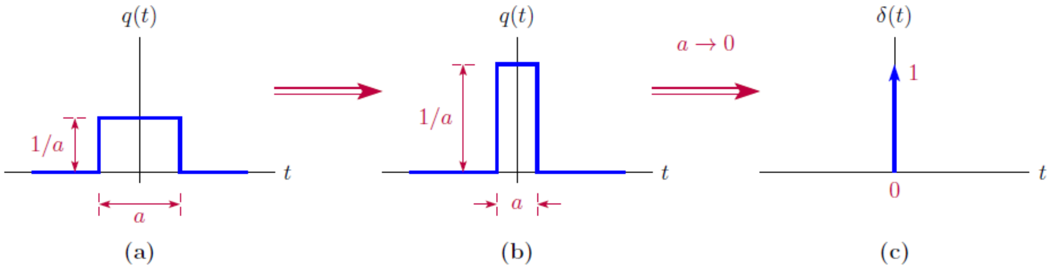
$$\int_{-\infty}^{\infty} a\,\delta(t - t_1)\ dt = a$$



1.3 Continuous-Time Signals

Obtaining unit-impulse function from a rectangular pulse

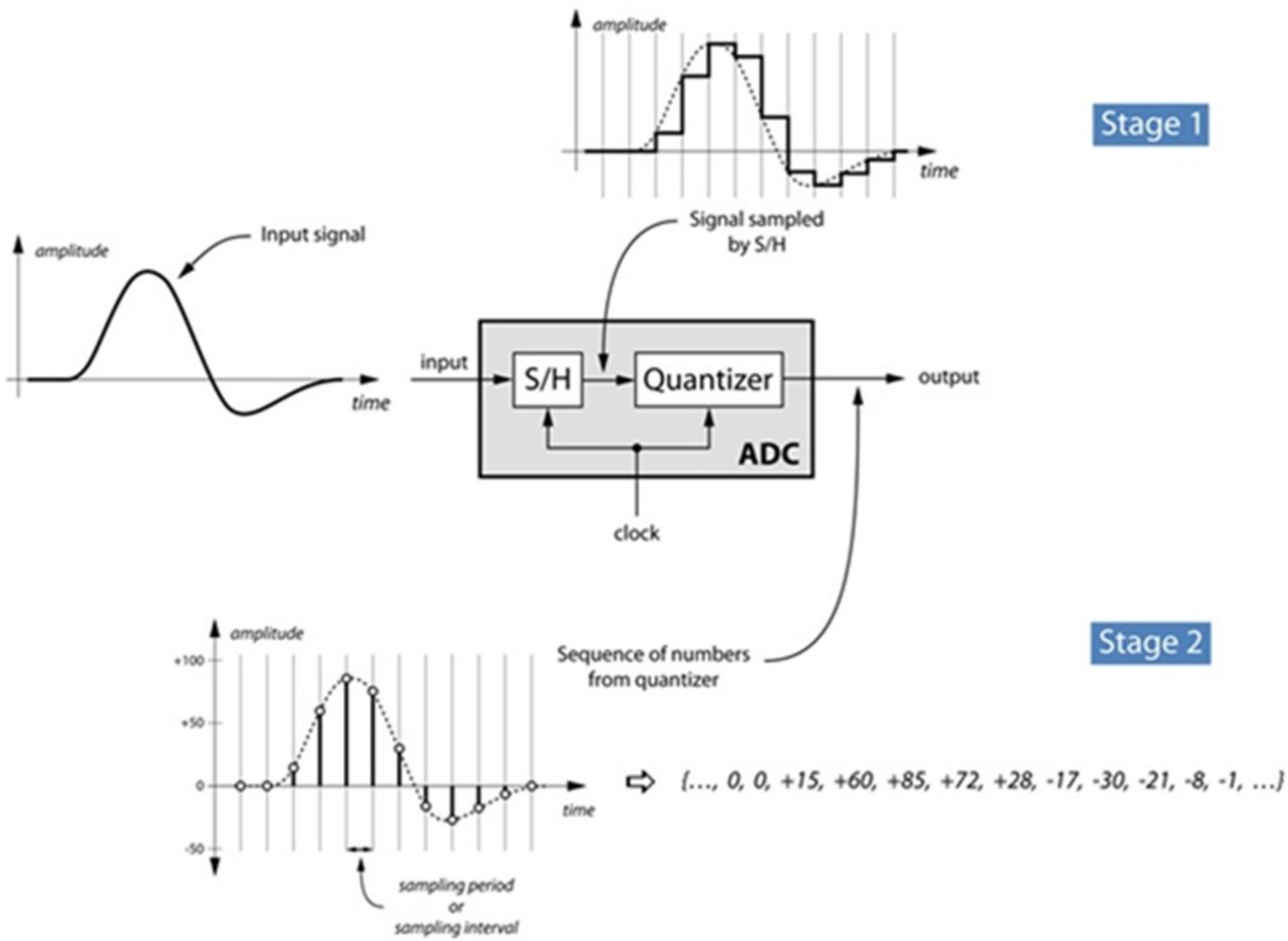
$$\text{Let} \quad q(t) = \begin{cases} \dfrac{1}{a}, & |t| < \dfrac{a}{2} \\ 0, & |t| > \dfrac{a}{2} \end{cases}$$



$$\delta(t) = \lim_{a \to 0} [q(t)]$$

As the pulse becomes narrower, it also becomes taller as shown in Figure. The area under the pulse remains unity. In the limit, as the parameter a approaches zero, the pulse approaches an impulse.

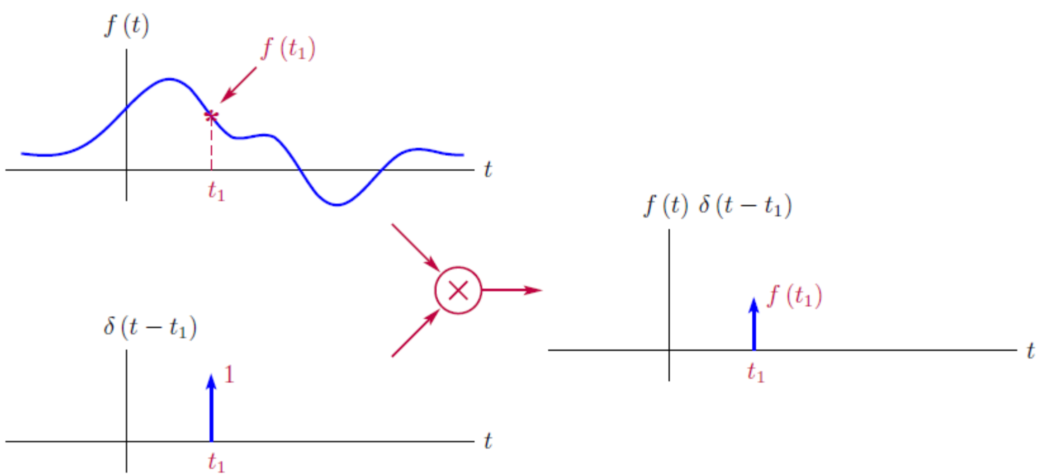
1.3 Continuous-Time Signals



1.3 Continuous-Time Signals

Sampling property of the unit-impulse function

$$f(t)\,\delta(t-t_1) = f(t_1)\,\delta(t-t_1)$$



The function $f(t)$ must be continuous at $t = t_1$.

The impulse function has two fundamental properties that are useful.
The first one, referred to as the *sampling property*

1.3 Continuous-Time Signals

Sifting property of the unit-impulse function

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-t_1)\,dt = f(t_1)$$

$$\int_{t_1-\Delta t}^{t_1+\Delta t} f(t)\,\delta(t-t_1)\,dt = f(t_1)$$

The function $f(t)$ must be continuous at $t = t_1$. Also, $\Delta t > 0$.

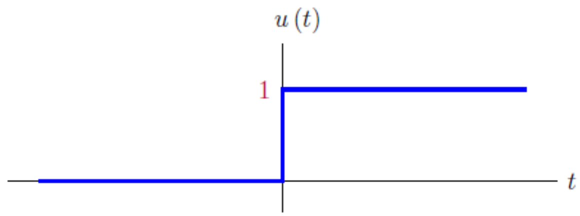
Shifting property: The integral of the product of a function *f(t)* and a time-shifted unit-impulse function is equal to the value of *f(t)* evaluated at the location of the unit impulse.

1.3 Continuous-Time Signals

Unit-step function

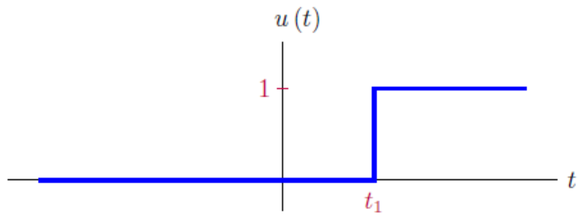
Mathematical definition

$$u(t) = \begin{cases} 1\,, & t > 0 \\ 0\,, & t < 0 \end{cases}$$



Time shifting the unit-step function:

$$u(t - t_1) = \begin{cases} 1\,, & t > t_1 \\ 0\,, & t < t_1 \end{cases}$$

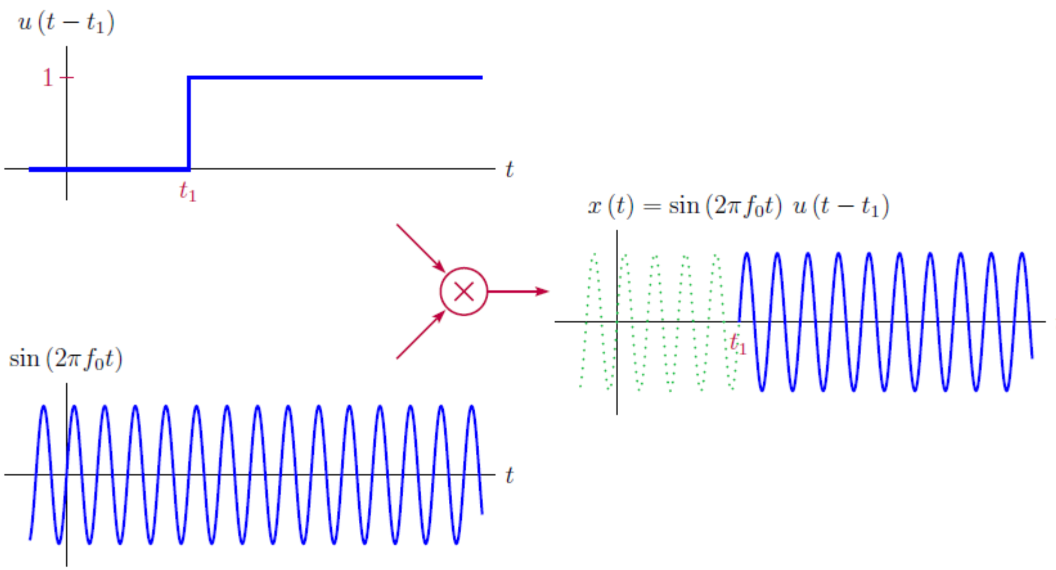


we need to model a signal that is *turned on or off* at a specific time instant.

1.3 Continuous-Time Signals

Using the unit-step function to turn a signal on

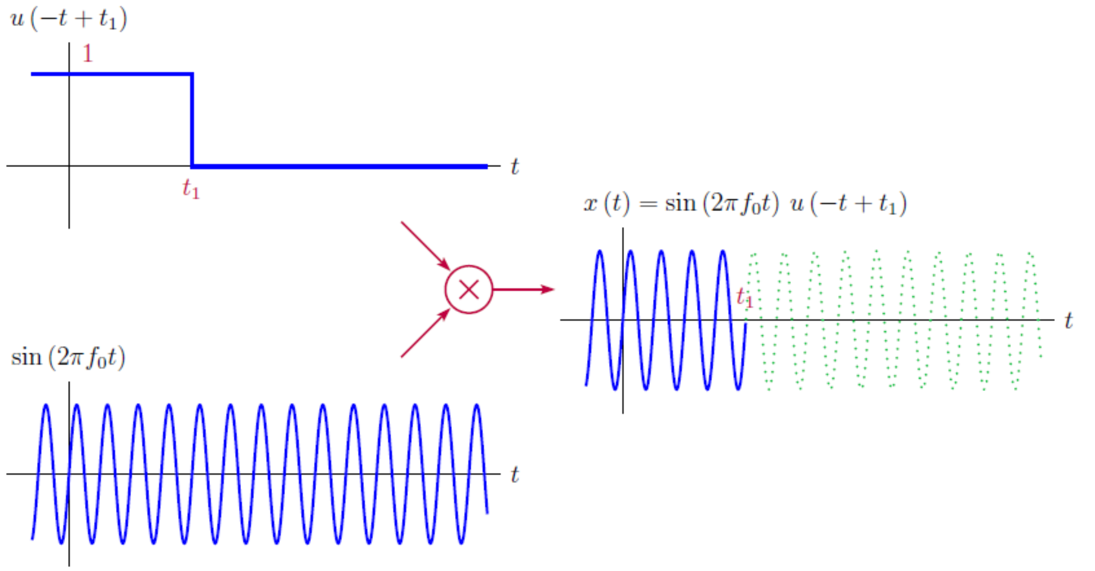
$$x(t) = \sin(2\pi f_0 t)\, u(t - t_1) = \begin{cases} \sin(2\pi f_0 t), & t > t_1 \\ 0, & t < t_1 \end{cases}$$



1.3 Continuous-Time Signals

Using the unit-step function to turn a signal off

$$x(t) = \sin(2\pi f_0 t)\; u(-t+t_1) = \begin{cases} \sin(2\pi f_0 t)\;, & t < t_1 \\ 0\;, & t > t_1 \end{cases}$$

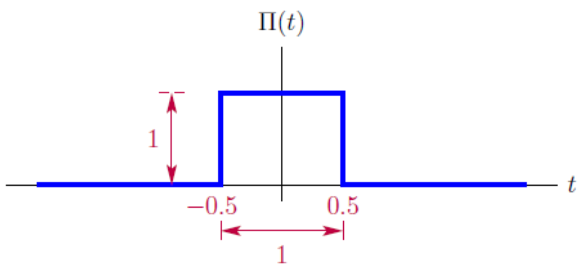


1.3 Continuous-Time Signals

Unit-pulse function

Mathematical definition

$$\Pi(t) = \begin{cases} 1, & |t| < \frac{1}{2} \\ 0, & |t| > \frac{1}{2} \end{cases}$$

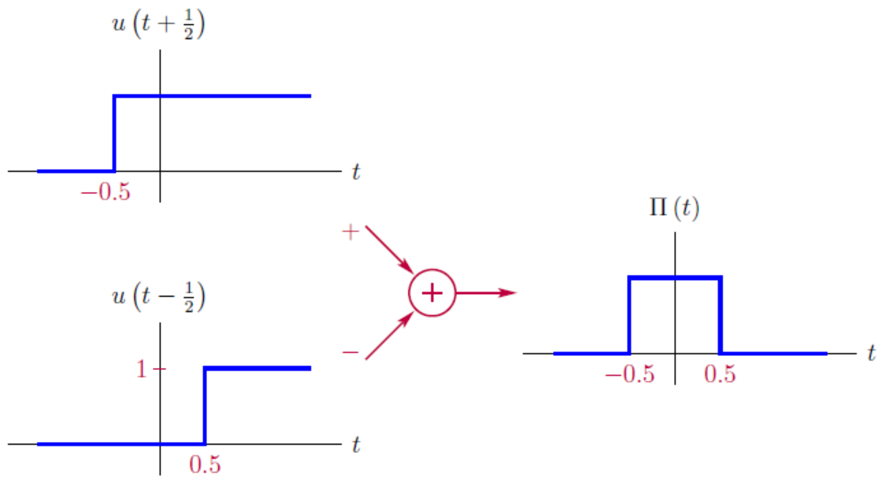


a rectangular pulse with unit width and unit amplitude, centered around the origin

1.3 Continuous-Time Signals

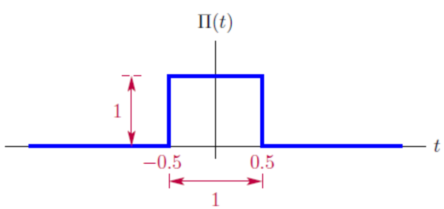
Constructing a unit-pulse from unit-step functions

$$\Pi(t) = u\left(t+\tfrac{1}{2}\right) - u\left(t-\tfrac{1}{2}\right)$$



Mathematical definition

$$\Pi(t) = \begin{cases} 1, & |t| < \frac{1}{2} \\ 0, & |t| > \frac{1}{2} \end{cases}$$

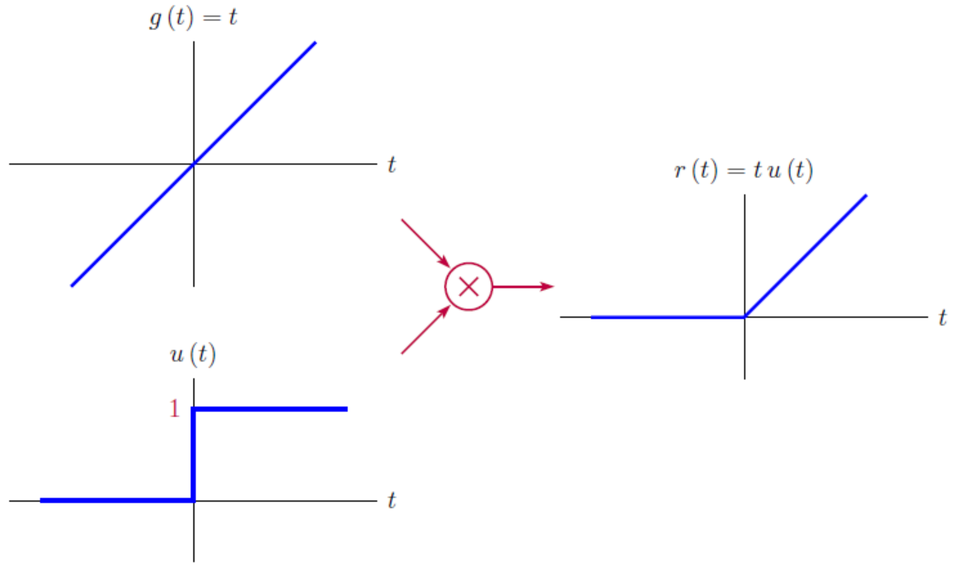


1.3 Continuous-Time Signals

Unit-ramp function

Mathematical definition

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases} \qquad \text{or, equivalently} \qquad r(t) = t\,u(t)$$

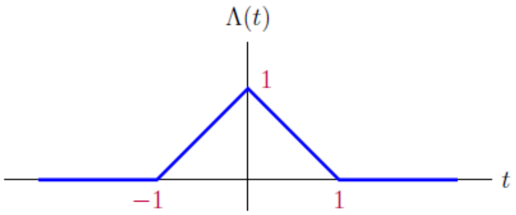


1.3 Continuous-Time Signals

Unit-triangle function

Mathematical definition

$$\Lambda(t) = \begin{cases} t+1, & -1 \leq t < 0 \\ -t+1, & 0 \leq t < 1 \\ 0, & \text{otherwise} \end{cases}$$



1.3 Continuous-Time Signals

Sinusoidal signals

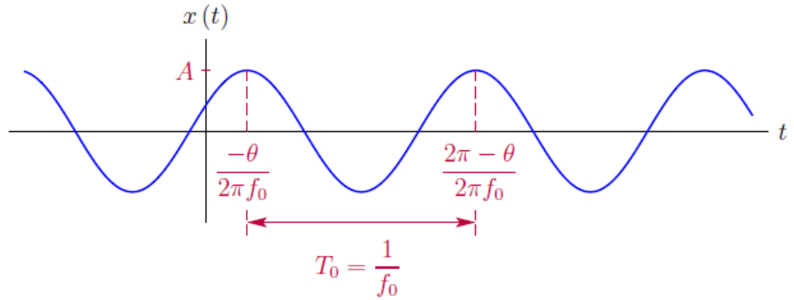
Sinusoidal signal

$$x(t) = A\cos(\omega_0 t + \theta)$$

A : Amplitude

ω_0 : Radian frequency (rad/s)

θ : Phase (radians)



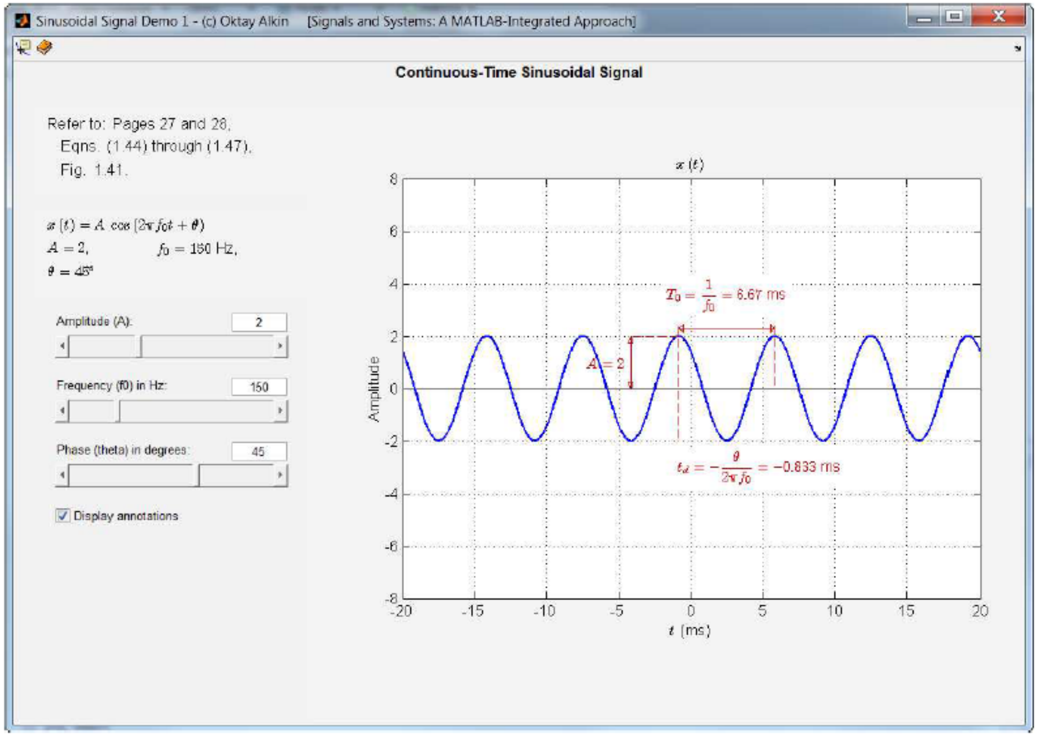
MATLAB Exercise 1.5

Cos(0) = 1 , -θ

1.3 Continuous-Time Signals

Interactive demo: sin_demo2

Experiment by varying the amplitude A, the frequency f_0 and the phase θ.



1.3 Continuous-Time Signals

Real vs. complex signals

Complex signal in Cartesian form

$$x\,(t) = x_r\,(t) + j\,x_i\,(t)$$

Complex signal in polar form

$$x\,(t) = |x\,(t)|\;e^{j\measuredangle x(t)}$$

$$|x(t)| = \left[x_r^2(t) + x_i^2(t)\right]^{1/2}$$

$$\measuredangle x(t) = \tan^{-1}\left[\frac{x_i(t)}{x_r(t)}\right]$$

$$x_r(t) = |x(t)|\;\cos\left(\measuredangle x(t)\right)$$

$$x_i(t) = |x(t)|\;\sin\left(\measuredangle x(t)\right)$$

1.3 Continuous-Time Signals

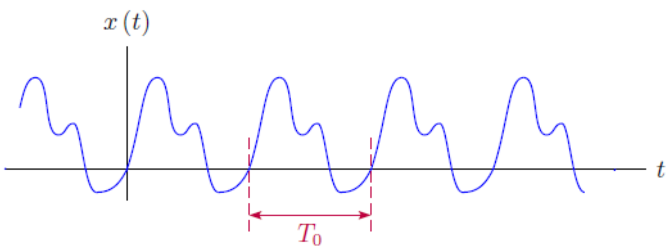
Periodic signals

Definition

A signal is said to be *periodic* if it satisfies

$$x(t + T_0) = x(t)$$

at all time instants t, and for a specific value of $T_0 \neq 0$.



If a signal is periodic with period T_0, then it is also periodic with periods of $2T_0, 3T_0, \ldots, kT_0, \ldots$ where k is any integer.

The ***fundamental frequency*** of a periodic signal is defined as the reciprocal of its fundamental period: $f_0 = 1 / T_0$

1.3 Continuous-Time Signals

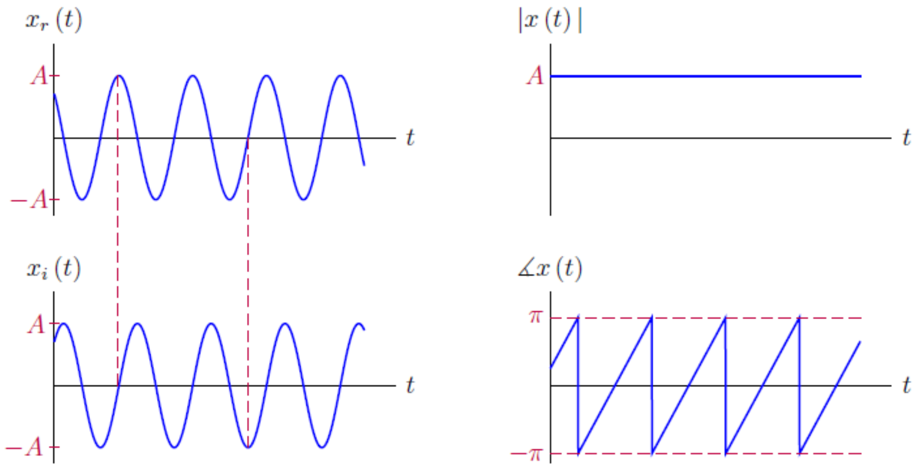
Example 1.4

Working with a complex periodic signal

Consider a signal defined by

$$\begin{aligned} x(t) &= x_r(t) + j\,x_i(t) \\ &= A\cos(2\pi f_0 t + \theta) + j\,A\sin(2\pi f_0 t + \theta) \end{aligned}$$

Graph the components in Cartesian and polar representations of this signal.



MATLAB Exercise 1.4

1.3 Continuous-Time Signals

Example 1.4: Working with a complex periodic signal

Consider a signal defined by

$$\begin{aligned} x(t) =& x_r(t) + x_i(t) \\ =& A\cos(2\pi f_0 t + \theta) + j\,A\sin(2\pi f_0 t + \theta) \end{aligned}$$

Using Eqns. (1.57) and (1.58), polar complex form of this signal can be obtained as $x(t) = |x(t)|\,e^{j\measuredangle x(t)}$, with magnitude and phase given by

$$|x(t)| = \left[\left[A\cos(2\pi f_0 t + \theta)\right]^2 + \left[A\sin(2\pi f_0 t + \theta)\right]^2\right]^{1/2} = A \tag{1.66}$$

and

$$\measuredangle x(t) = \tan^{-1}\left[\frac{\sin(2\pi f_0 t + \theta)}{\cos(2\pi f_0 t + \theta)}\right] = \tan^{-1}\left[\tan(2\pi f_0 t + \theta)\right] = 2\pi f_0 t + \theta \tag{1.67}$$

respectively. In deriving the results in Eqns. (1.66) and (1.67) we have relied on the appropriate trigonometric identities.[4] Once the magnitude $|x(t)|$ and the phase $\measuredangle x(t)$ are obtained, we can express the signal $x(t)$ in polar complex form:

$$x(t) = |x(t)|\,e^{j\measuredangle x(t)} = A\,e^{j(2\pi f_0 t + \theta)} \tag{1.68}$$

The components of the Cartesian and polar complex forms of the signal are shown in Fig. 1.44. The real and imaginary parts of $x(t)$ have a 90 degree phase difference between them. When the real part of the signal goes through a peak, the imaginary part goes through zero and vice versa. The phase of $x(t)$ was found in Eqn. (1.66) to be a linear function of the

[4] $\cos^2(a) + \sin^2(a) = 1$, and $\tan(a) = \sin(a)/\cos(a)$.

1.3 Continuous-Time Signals

Deterministic vs. Random signals

Deterministic signals are those that can be described completely in the analytical form in the time domain.

Random signals, on the other hand, are signals that occur due to random phenomena that cannot be modeled analytically.
An example of a random signal is the vibration signal recorded during an earthquake by a seismograph.

1.3 Continuous-Time Signals

Energy computations

Normalized energy of a signal

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt$$

if the integral can be computed.

Normalized energy of a complex signal

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

if the integral can be computed.

$$E = \int_{-\infty}^{\infty} v(t)\,i(t)\,dt = \int_{-\infty}^{\infty} \frac{v^2(t)}{R}\,dt$$

$$E = \int_{-\infty}^{\infty} v(t)\,i(t)\,dt = \int_{-\infty}^{\infty} R\,i^2(t)\,dt$$

With physical signals and systems, the concept of energy is associated with a ***signal that is applied to a load***.
The signal source delivers the energy which must be ***dissipated by the load***.

1.3 Continuous-Time Signals

Time averaging operator

Time average of a signal periodic with period T_0

$$\langle x(t) \rangle = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t)\, dt$$

Time average of an aperiodic signal

$$\langle x(t) \rangle = \lim_{T \to \infty} \left[\frac{1}{T} \int_{-T/2}^{T/2} x(t)\, dt \right]$$

We will use the operator $< x(t) >$ to indicate the time average.

1.3 Continuous-Time Signals

Power computations

Normalized instantaneous power (real signal)

$$p_{\text{norm}}(t) = x^2(t)$$

Normalized instantaneous power (complex signal)

$$p_{\text{norm}}(t) = |x(t)|^2$$

Normalized average power (real signal)

$$P_x = \left\langle x^2(t) \right\rangle$$

Normalized average power (complex signal)

$$P_x = \left\langle |x(t)|^2 \right\rangle$$

1.3 Continuous-Time Signals

Energy signals vs. power signals

- Energy signals are those that have finite energy, and zero power. $E_x < \infty$, and $P_x = 0$.
- Power signals are those that have finite power and infinite energy. $E_x \to \infty$, and $P_x < \infty$.

all voltage and current signals that can be generated in the laboratory or that occur in the electronic devices that we use in our daily lives are ***energy signals***.

A power signal is impossible to produce in any practical setting since doing so would require an infinite amount of energy. The concept of a power signal exists as a mathematical idealization only.

1.3 Continuous-Time Signals

Symmetry properties

Even symmetry

A real-valued signal is said to have *even symmetry* if it has the property

$$x(-t) = x(t)$$

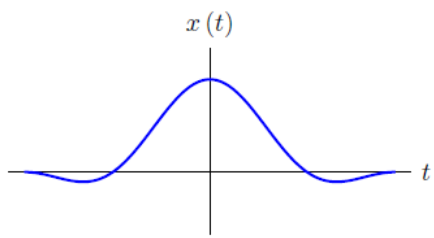
for all values of t.

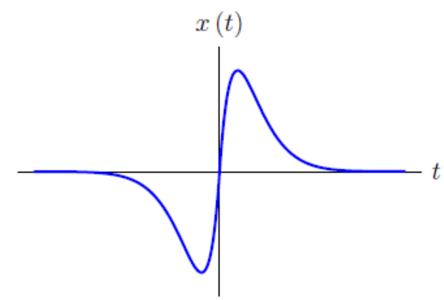
Odd symmetry

A real-valued signal is said to have *odd symmetry* if it has the property

$$x(-t) = -x(t)$$

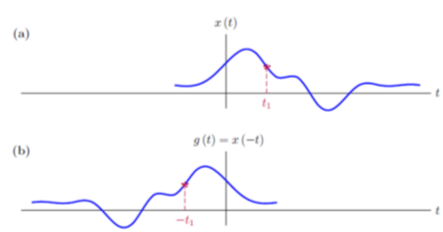
for all values of t.





Time reversal

$$g(t) = x(-t)$$



Symmetry properties

1.3 Continuous-Time Signals

Graphical representation of sinusoidal signals using phasors

$$x(t) = A\cos(2\pi f_0 t + \theta)$$

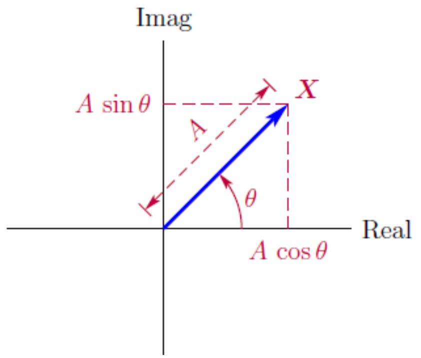
Let the phasor X be defined as

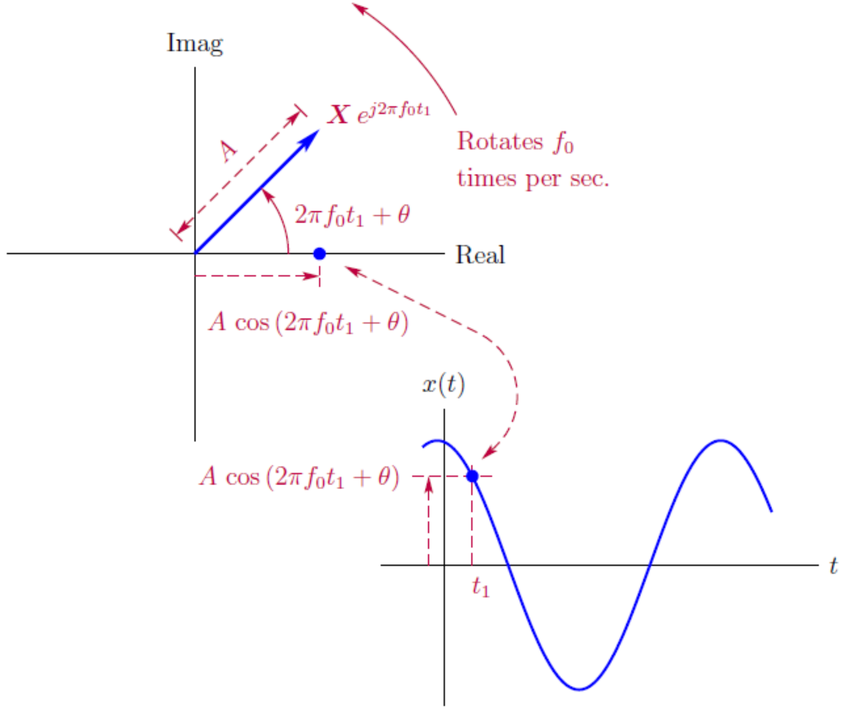
$$X \triangleq A\,e^{j\theta}$$

so that

$$x(t) = \mathrm{Re}\left\{ A\,e^{j(2\pi f_0 t + \theta)} \right\}$$

$$= \mathrm{Re}\left\{ X\,e^{j2\pi f_0 t} \right\}$$



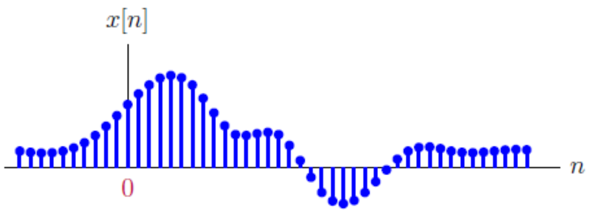


[e^(iπ) in 3.14 minutes]

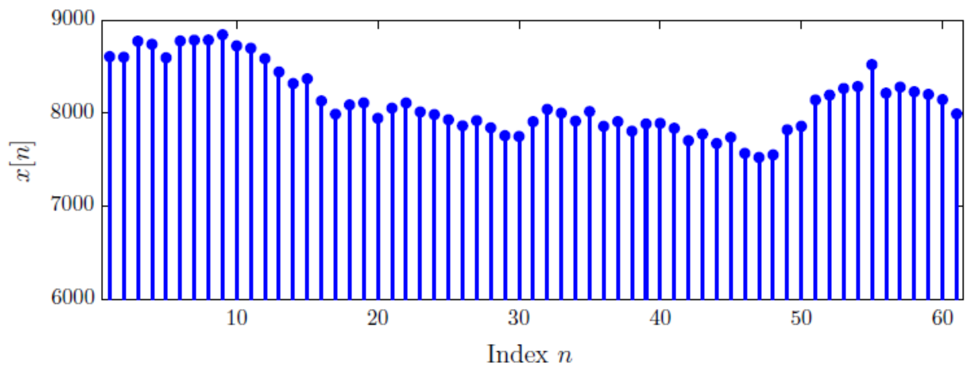
1.4 Discrete-Time Signals

Discrete-time signals

A discrete-time signal.



Dow Jones Industrial Average for the first three months of 2003.



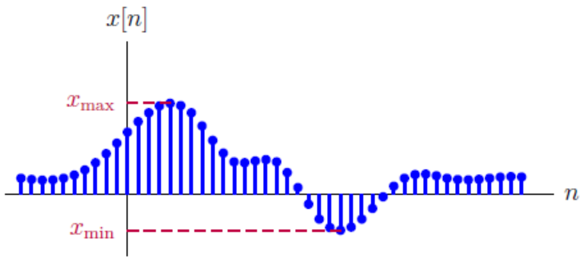
Discrete-time signals are not defined ***at all time instants***. Instead, they are defined only ***at time instants*** that are integer multiples of a fixed time increment T, that is, at t = nT.

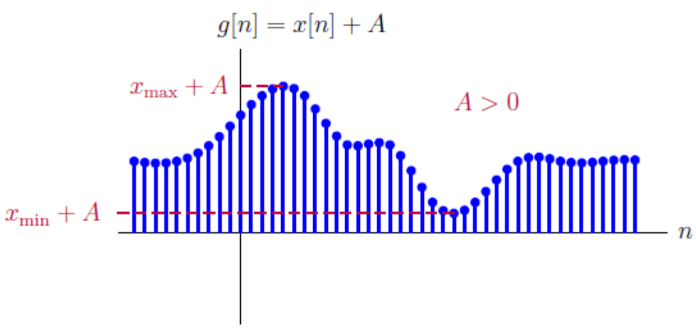
1.4 Discrete-Time Signals

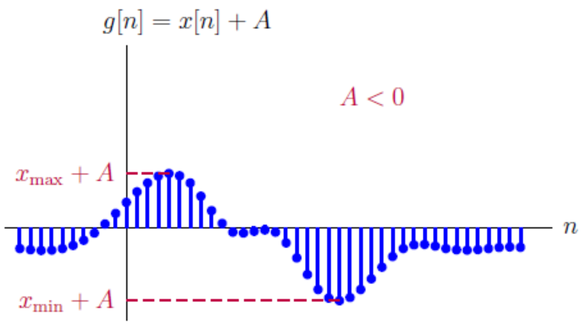
Signal operations

Addition of a constant offset

$$g[n] = x[n] + A$$







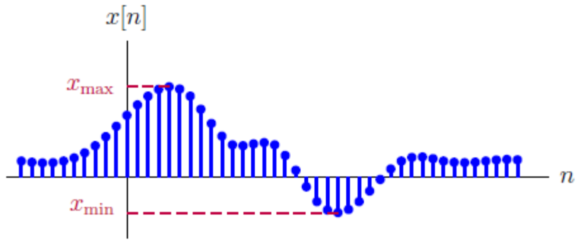
Signal Operation

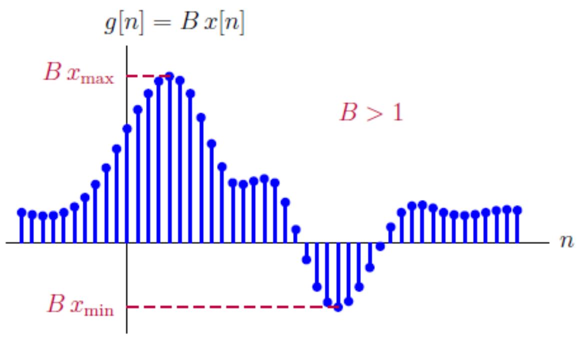
1.4 Discrete-Time Signals

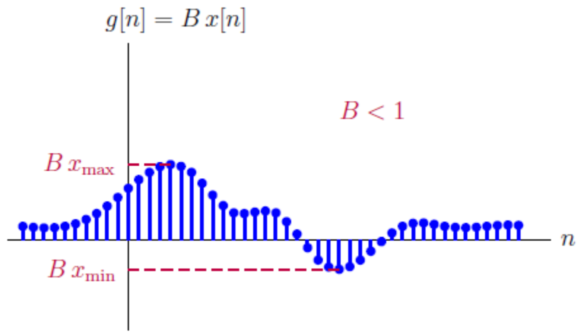
Signal operations (continued)

Multiplication by a constant gain factor

$$g[n] = B\, x[n]$$





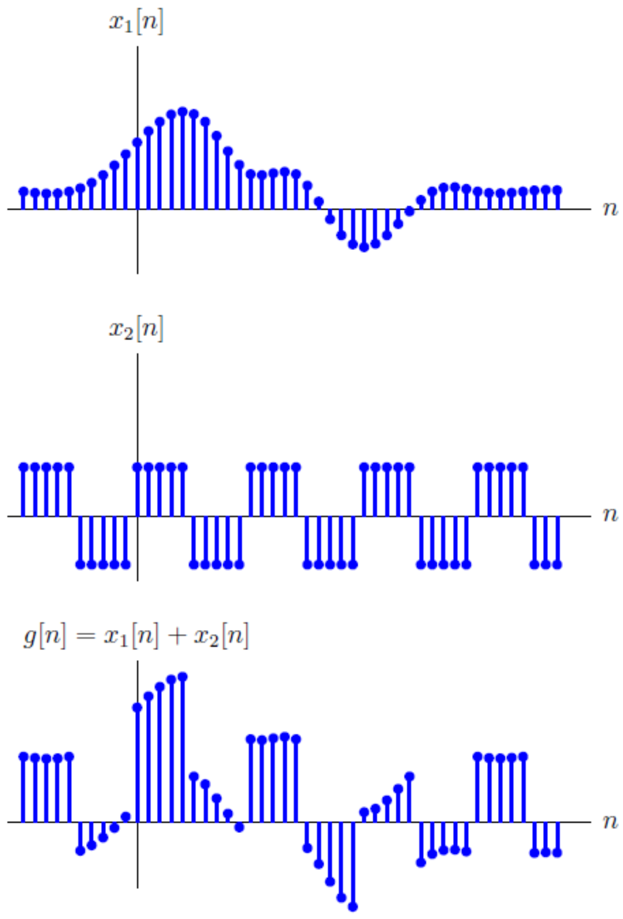


1.4 Discrete-Time Signals

Signal operations (continued)

Adding two signals

$$g[n] = x_1[n] + x_2[n]$$

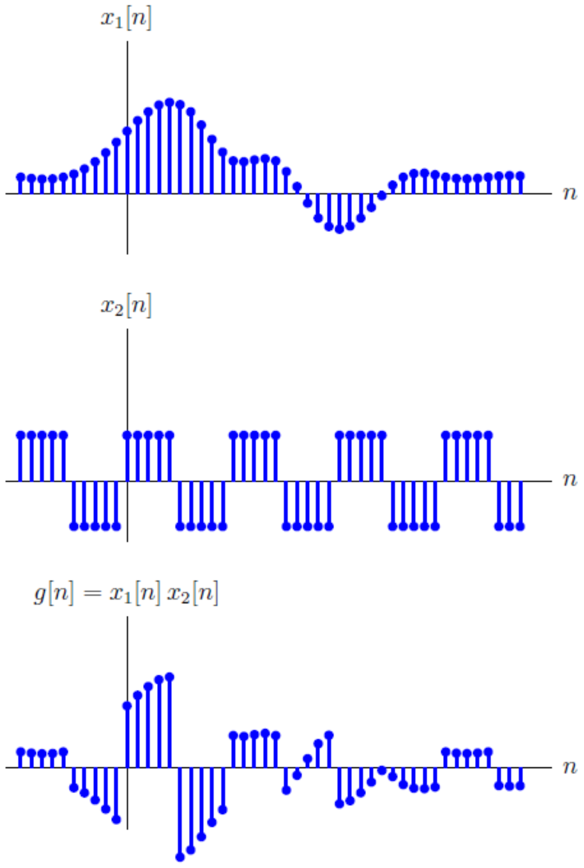


1.4 Discrete-Time Signals

Signal operations (continued)

Multiplying two signals

$$g[n] = x_1[n]\,x_2[n]$$



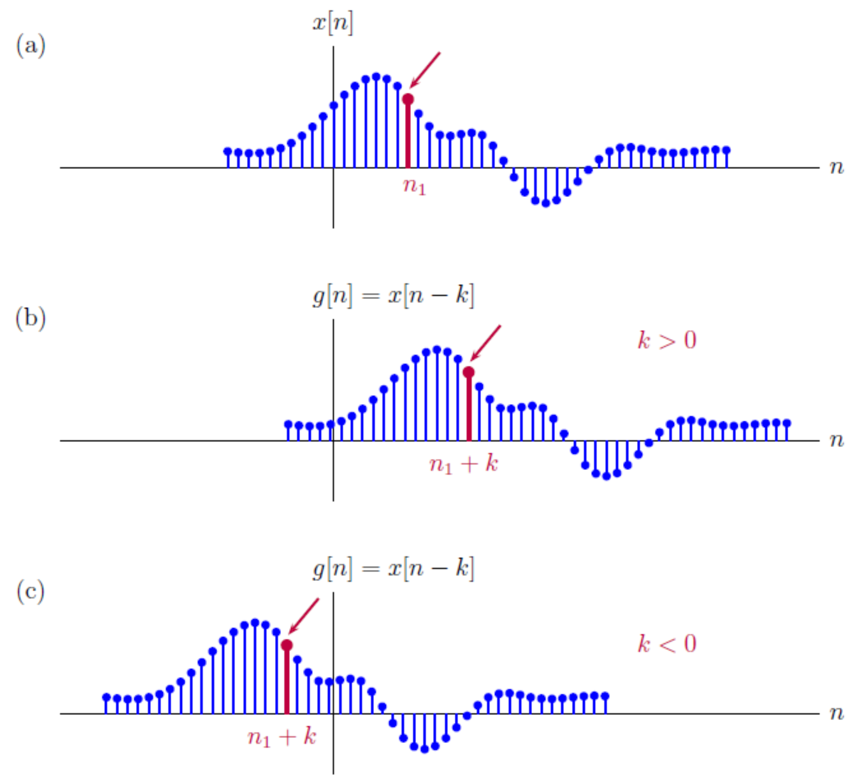
1.4 Discrete-Time Signals

Signal operations (continued)

Time shifting

$$g[n] = x[n-k]$$

k: Integer



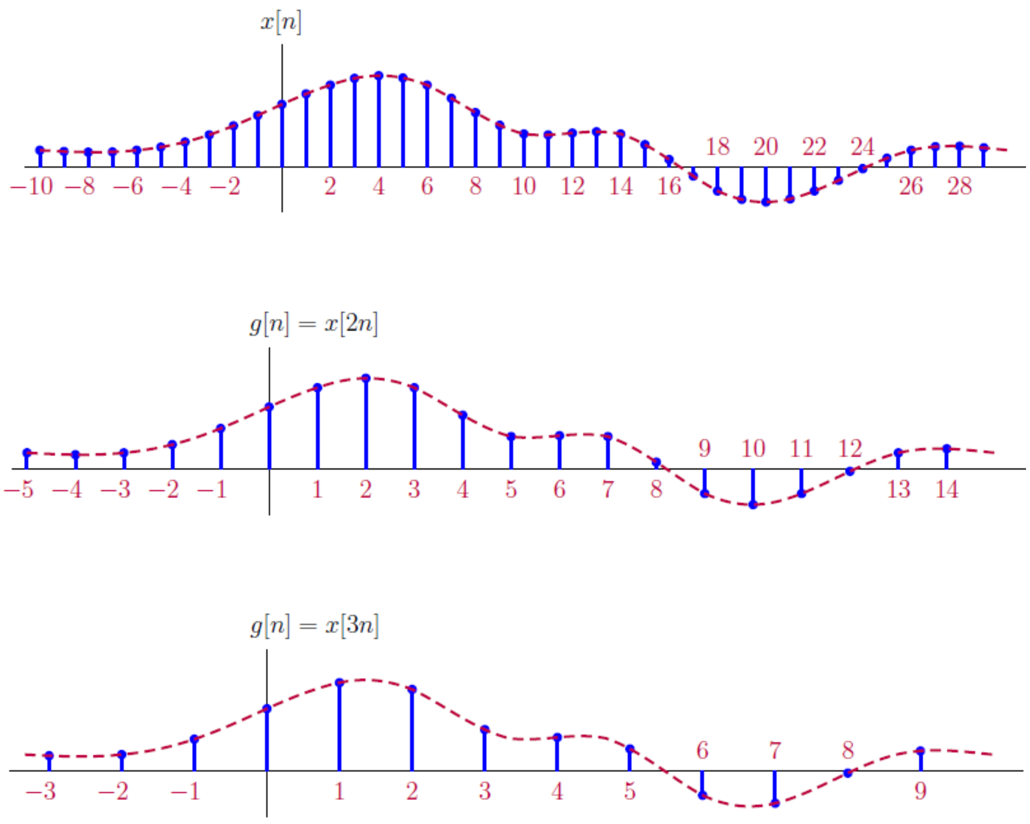
1.4 Discrete-Time Signals

Signal operations (continued)

Time scaling

$$g[n] = x[kn]$$

k: integer

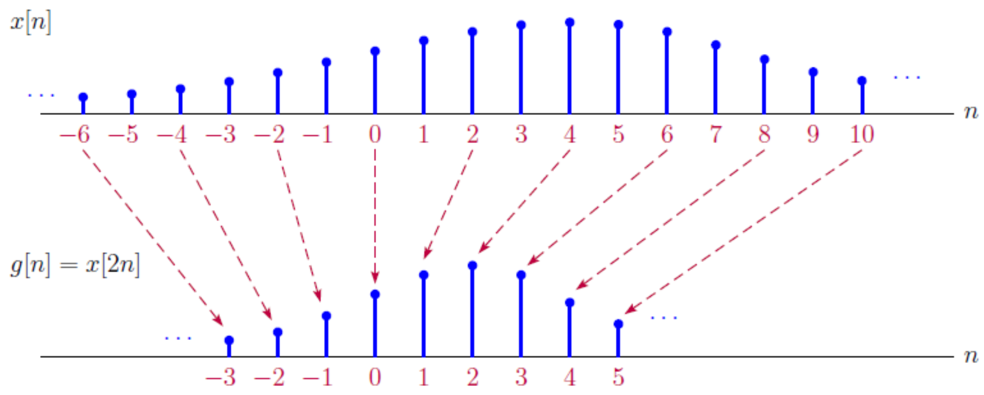


1.4 Discrete-Time Signals

Signal operations (continued)

Time scaling example (downsampling)

$$g[n] = x[2n]$$

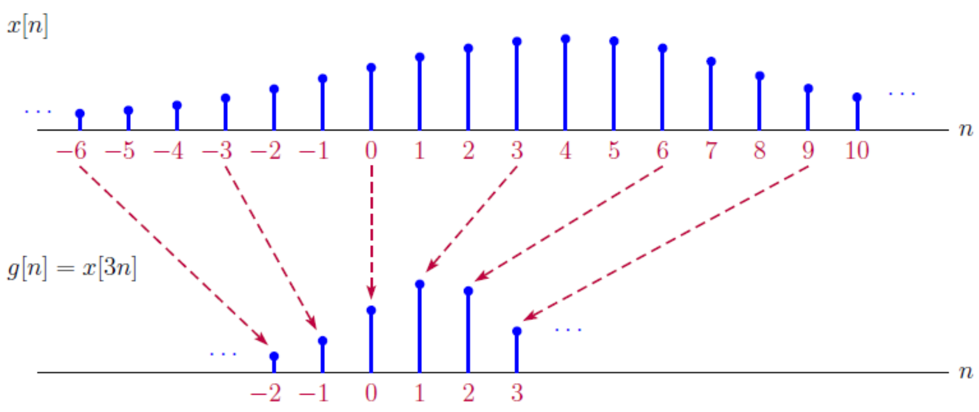


1.4 Discrete-Time Signals

Signal operations (continued)

Time scaling example (downsampling)

$$g[n] = x[3n]$$



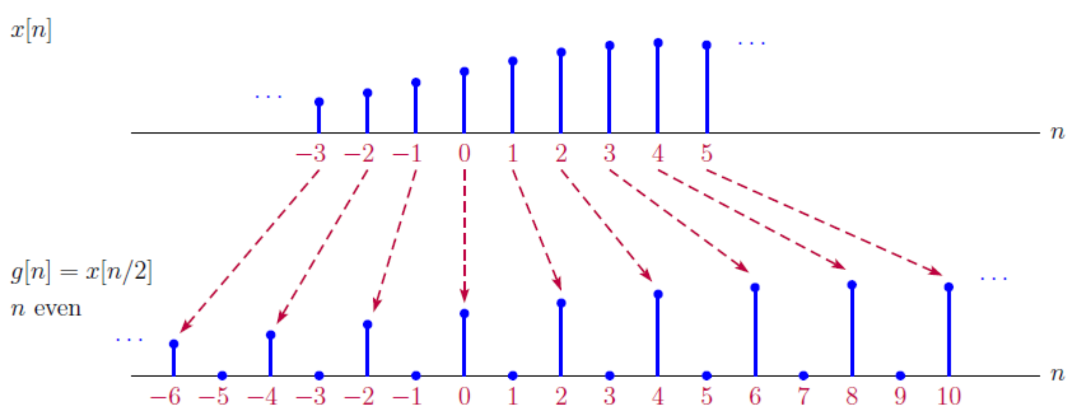
1.4 Discrete-Time Signals

Signal operations (continued)

An alternative form of time scaling (upsampling)

$g[n] = x[n/2]$ How do we handle odd values of n?

$$g[n] = \begin{cases} x[n/2]\,, & \text{if } n/2 \text{ is integer} \\ 0\,, & \text{otherwise} \end{cases}$$

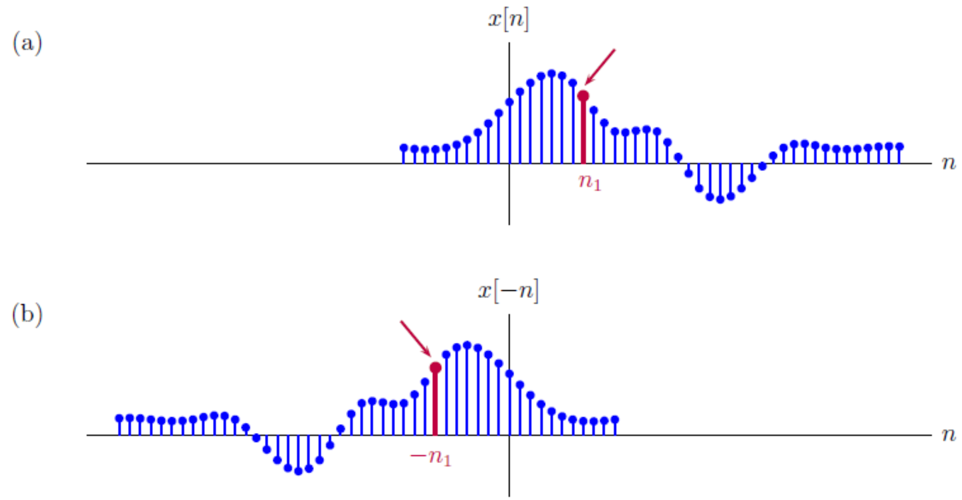


1.4 Discrete-Time Signals

Signal operations (continued)

Time reversal

$$g[n] = x[-n]$$



1.4 Discrete-Time Signals

Basic building blocks

- Unit-impulse function
- Unit-step function
- Unit-ramp function
- Sinusoidal signals

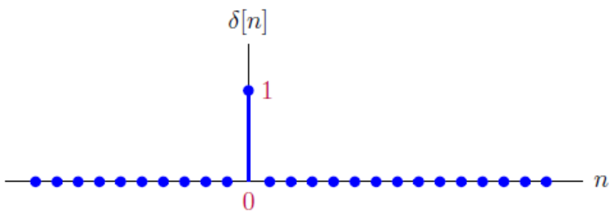
Basic building blocks

1.4 Discrete-Time Signals

Unit-impulse function

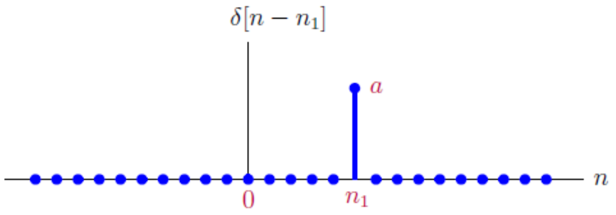
Mathematical definition

$$\delta[n] = \begin{cases} 1\,, & n = 0 \\ 0\,, & n \neq 0 \end{cases}$$



Scaling and time shifting:

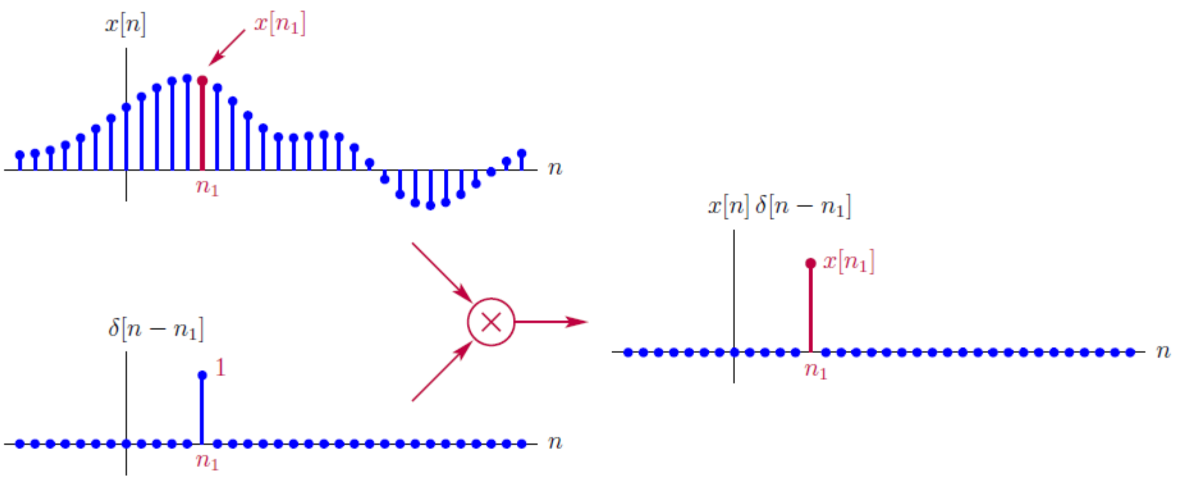
$$a\,\delta[n - n_1] = \begin{cases} a\,, & n = n_1 \\ 0\,, & n \neq n_1 \end{cases}$$



1.4 Discrete-Time Signals

Sampling property of the unit-impulse function

$$x[n]\,\delta[n-n_1] = x[n_1]\,\delta[n-n_1]$$



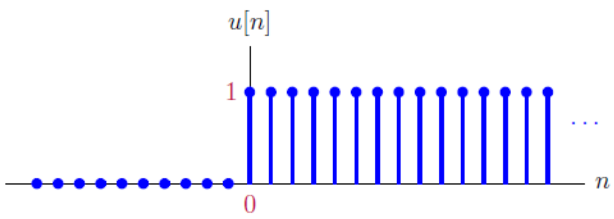
$$x[n]\,\delta[n-n_1] = \begin{cases} x[n_1]\,, & n = n_1 \\ 0\,, & n \neq n_1 \end{cases}$$

1.4 Discrete-Time Signals

Unit-step function

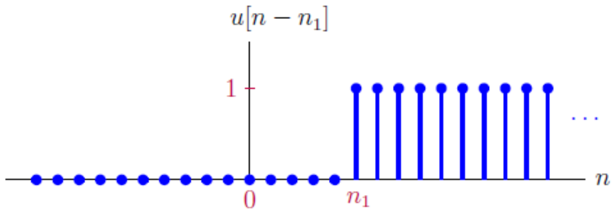
Mathematical definition

$$u[n] = \begin{cases} 1\,, & n \geq 0 \\ 0\,, & n < 0 \end{cases}$$



Time shifting the unit-step function:

$$u[n - n_1] = \begin{cases} 1\,, & n \geq n_1 \\ 0\,, & n < n_1 \end{cases}$$



1.4 Discrete-Time Signals

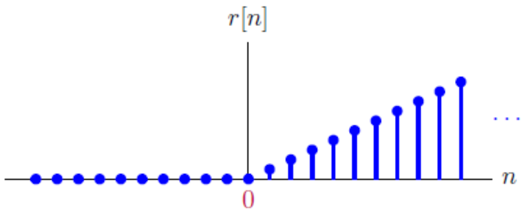
Unit-ramp function

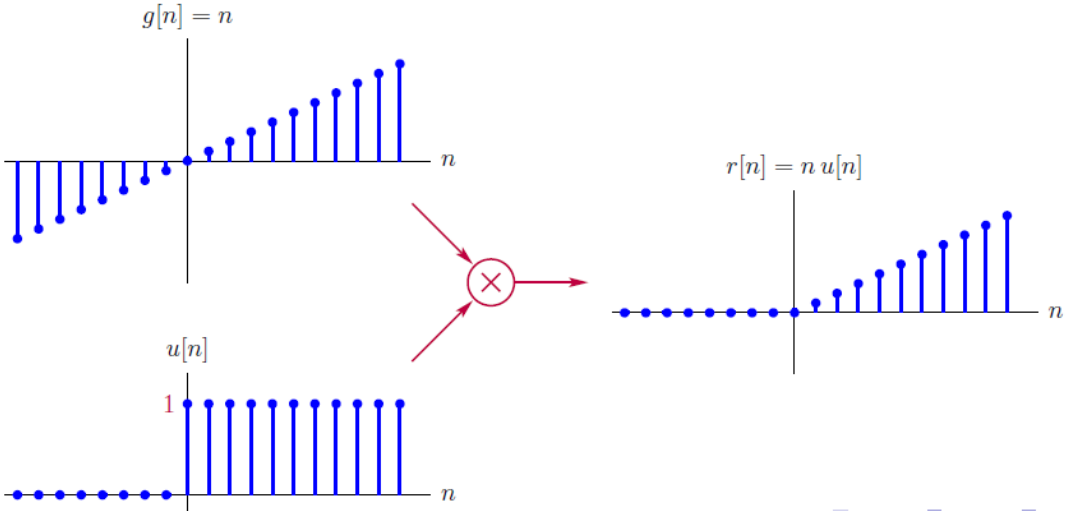
Mathematical definition

$$r[n] = \begin{cases} n\,, & n \geq 0 \\ 0\,, & n < 0 \end{cases}$$

or, equivalently

$$r[n] = n\,u[n]$$





1.4 Discrete-Time Signals

Sinusoidal signals

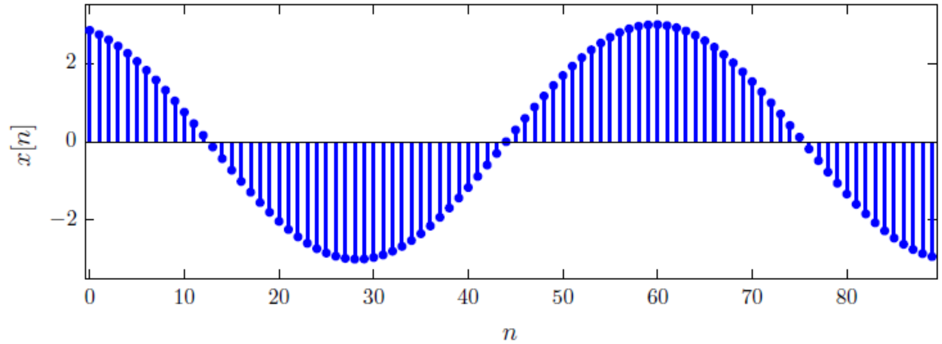
Sinusoidal signal

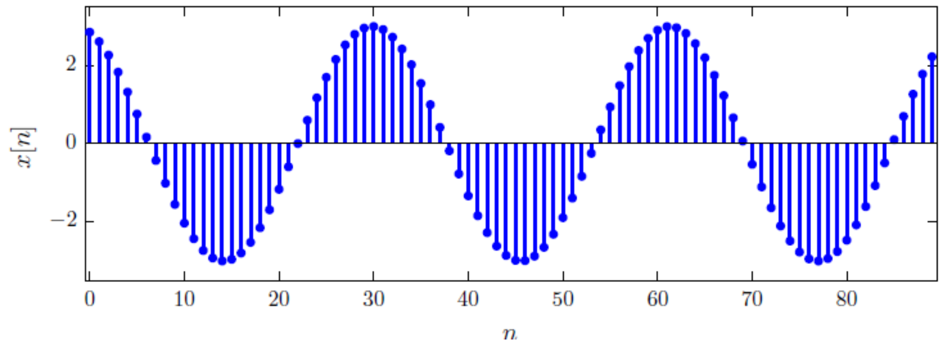
$$x[n] = A\cos(\Omega_0 n + \theta)$$

A : Amplitude

Ω_0 : Angular frequency (radians)

θ : Phase (radians)





1.4 Discrete-Time Signals

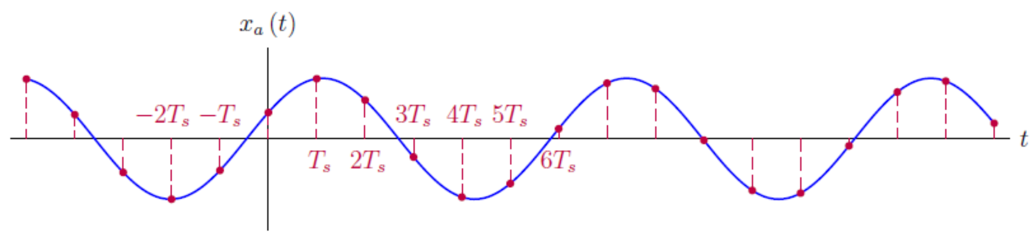
Characteristics of discrete-time sinusoids

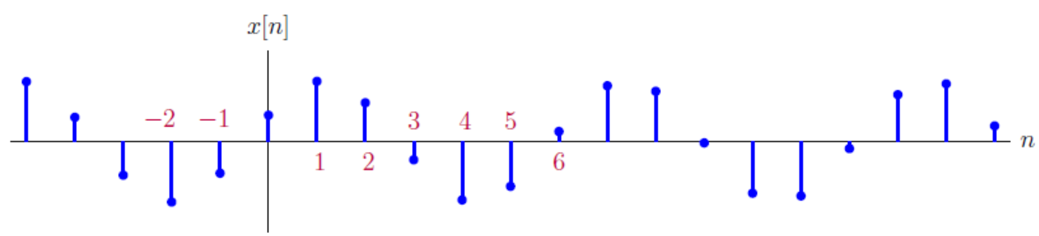
- For continuous-time sinusoidal signal $x_a(t) = A\cos(\omega_0 t)$: ω_0 is in rad/s.
- For discrete-time sinusoidal signal $x[n] = A\cos(\Omega_0 n)$: Ω_0 is in radians.

$$x_a(t) = A\cos(\omega_0 t + \theta)$$

$$\begin{aligned} x[n] &= x_a(nT_s) \\ &= A\cos(\omega_0 T_s n + \theta) \\ &= A\cos(2\pi f_0 T_s n + \theta) \end{aligned}$$

$$\begin{aligned} \Omega_0 &= \omega_0 T_s \\ F_0 &= f_0 T_s \\ \Omega_0 &= 2\pi F_0 \end{aligned}$$





1.4 Discrete-Time Signals

Real vs. complex signals

Complex signal in Cartesian form

$$x[n] = x_r[n] + j\, x_i[n]$$

Complex signal in polar form

$$x[n] = |x[n]|\; e^{j\measuredangle x[n]}$$

$$|x[n]| = \left(x_r^2[n] + x_i^2[n]\right)^{1/2}$$

$$\measuredangle x[n] = \tan^{-1}\left(\frac{x_i[n]}{x_r[n]}\right)$$

$$x_r[n] = |x[n]|\; \cos\left(\measuredangle x[n]\right)$$

$$x_i[n] = |x[n]|\; \sin\left(\measuredangle x[n]\right)$$

1.4 Discrete-Time Signals

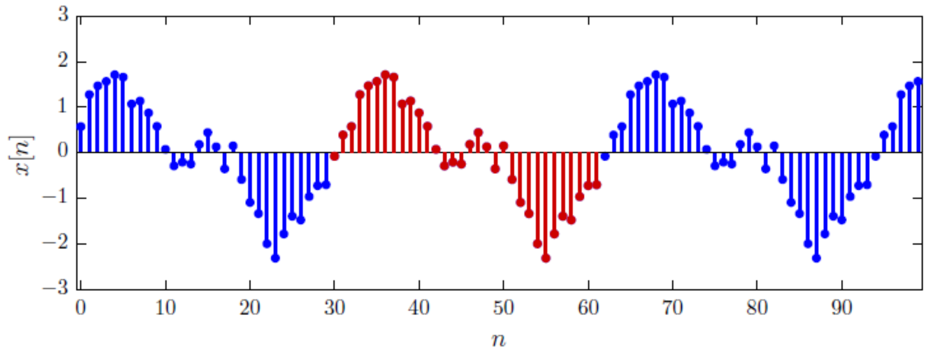
Periodic signals

Definition

A signal is said to be *periodic* if it satisfies

$$x[n+N] = x[n]$$

for all integer n and for a specific value of $N \neq 0$.



MATLAB Exercise 1.7

If a signal is periodic with period N, then it is also periodic with periods of $2N, 2N, \ldots, kN, \ldots$ where k is any integer.

1.4 Discrete-Time Signals

Energy computations

Normalized energy of a signal

$$E_x = \sum_{-\infty}^{\infty} x^2[n]$$

if the result of the summation can be computed.

Normalized energy of a complex signal

$$E_x = \sum_{-\infty}^{\infty} |x[n]|^2$$

if the result of the summation can be computed.

1.4 Discrete-Time Signals

Power computations

Normalized avg. power (real signal)

$$P_x = \left\langle x^2[n] \right\rangle$$

Normalized avg. power (complex signal)

$$P_x = \left\langle |x[n]|^2 \right\rangle$$

Periodic signal:

$$P_x = \frac{1}{N} \sum_{n=0}^{N-1} x^2[n]$$

Non-periodic signal:

$$P_x = \lim_{M \to \infty} \left[\frac{1}{2M+1} \sum_{n=-M}^{M} x^2[n] \right]$$

Periodic signal:

$$P_x = \frac{1}{N} \sum_{n=0}^{N-1} \left| x[n] \right|^2$$

Non-periodic signal:

$$P_x = \lim_{M \to \infty} \left[\frac{1}{2M+1} \sum_{n=-M}^{M} \left| x[n] \right|^2 \right]$$

1.4 Discrete-Time Signals

Symmetry properties

Even symmetry

A real-valued signal is said to have *even symmetry* if it has the property

$$x[-n] = x[n]$$

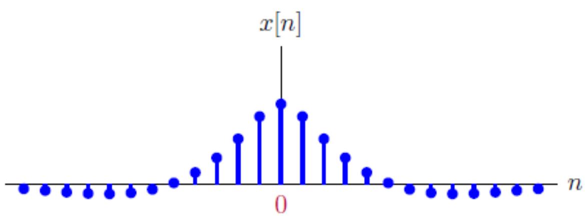
for all integer values of n.

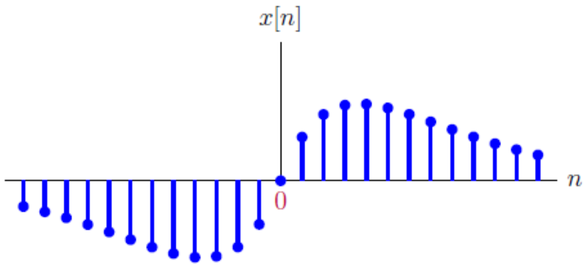
Odd symmetry

A real-valued signal is said to have *odd symmetry* if it has the property

$$x[-n] = -x[n]$$

for all integer values of n.





Symmetry properties

1.4 Discrete-Time Signals

MATLAB Exercise 1.6

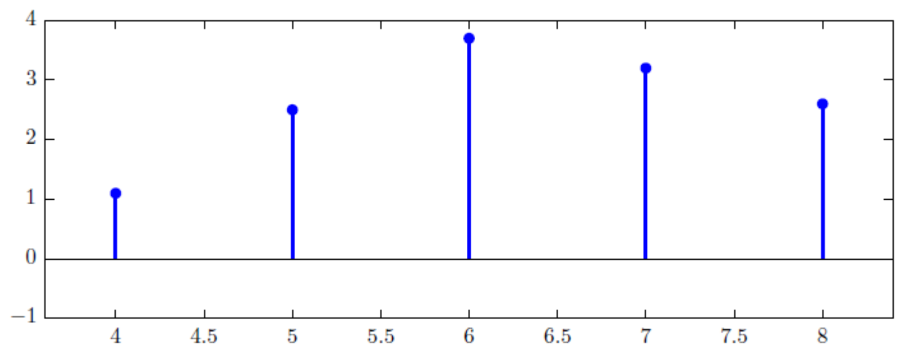
Part (a)

Compute and graph the signal

$$x_1[n] = \{ 1.1,\ \underset{\underset{n=5}{\uparrow}}{2.5},\ 3.7,\ 3.2,\ 2.6 \}$$

for the index range $4 \le n \le 8$.

```
n = [4:8];
x1 = [1.1,2.5,3.7,3.2,2.6];
stem(n,x1);
```



Conclusion

1 Signal Representation and Modeling

Chapter Objectives . . . .

1.1 Introduction . . . .

1.2 Mathematical Modeling of Signals . . . .

1.3 Continuous-Time Signals . . . .

1.3.1 Signal operations . . . .

1.3.2 Basic building blocks for continuous-time signals . . . .

1.3.3 Impulse decomposition for continuous-time signals . . . .

1.3.4 Signal classifications . . . .

1.3.5 Energy and power definitions . . . .

1.3.6 Symmetry properties . . . .

1.3.7 Graphical representation of sinusoidal signals using phasors . . . .

1.4 Discrete-Time Signals . . . .

1.4.1 Signal operations . . . .

1.4.2 Basic building blocks for discrete-time signals . . . .

1.4.3 Impulse decomposition for discrete-time signals . . . .

1.4.4 Signal classifications . . . .

1.4.5 Energy and power definitions . . . .

1.4.6 Symmetry properties . . . .